

# 1 Functions and rates of change – Part 1

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

## Vocabulary

*Write each word three times.*

| English                   | Write it 3 times (English) |       |       |
|---------------------------|----------------------------|-------|-------|
| function                  | _____                      | _____ | _____ |
| domain                    | _____                      | _____ | _____ |
| range                     | _____                      | _____ | _____ |
| increasing                | _____                      | _____ | _____ |
| decreasing                | _____                      | _____ | _____ |
| average rate of<br>change | _____                      | _____ | _____ |
| secant line               | _____                      | _____ | _____ |
| linear                    | _____                      | _____ | _____ |
| quadratic                 | _____                      | _____ | _____ |

## Recall

You have already worked with functions and graphs:

- A function turns each input into one output, like  $f(x) = x^2$ .
- You have found the gradient (slope) of a straight line.
- You know a quadratic makes a U-shaped parabola.

This sheet lines up how functions change. Each idea has a worked example.

## New ideas

### ■ 1 Functions, domain, and range

A **function** gives each input exactly one output. The allowed inputs are the **domain**; the outputs are the **range**.

### ■ 2 Increasing and decreasing

A function is **increasing** where a bigger input gives a bigger output, and **decreasing** where a bigger input gives a smaller output.

### ■ 3 Average rate of change

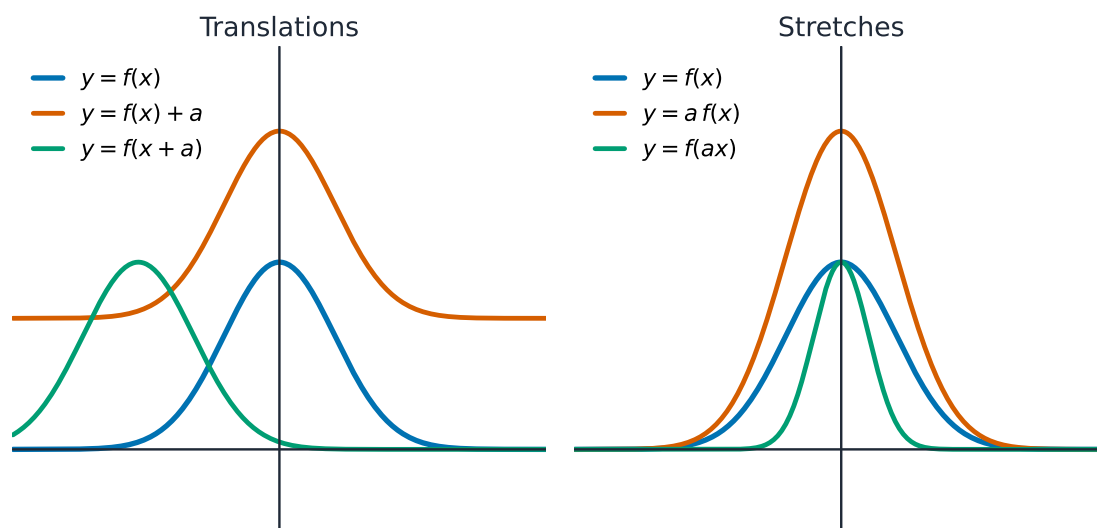
The **average rate of change** over an interval  $[a, b]$  is the change in output over the change in input:

$$\text{avg rate} = \frac{f(b) - f(a)}{b - a}.$$

This is just the gradient of the straight line (the **secant line**) joining the two points.

*Worked example.* For  $f(x) = x^2$  over  $[1, 4]$ :  $\frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = 5$ .

### ■ 4 Linear versus quadratic



For a **linear** function the average rate of change is **constant**; for a **quadratic** function it **changes** from interval to interval.

**Watch out:** the average rate of change is the slope of the line between two points – so for a curve it depends on **which** two points you pick. Only a straight line has the same rate everywhere.

## Practice

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Try these in order. The methods are all above.

**2.1** State what the domain and the range of a function are. [2]

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**2.2** For  $f(x) = 3x + 1$ , find the average rate of change over  $[0, 4]$ . [2]

**2.3** For  $f(x) = x^2$ , find the average rate of change over  $[2, 5]$ . [3]

**2.4** State whether  $f(x) = x^2$  is increasing or decreasing for  $x > 0$ . [1]

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**2.5** Explain why a linear function has the same average rate of change over every interval. [2]

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