4.12 Linear Transformations and Matrices

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS linear transformation • matrix-vector multiplication • matrix • vector

Objective

Build the skills to answer exam questions on **linear transformations and matrices**.

You must be able to:

- apply a matrix to a vector by **matrix-vector multiplication**
- describe the geometric effect (scaling, identity)

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Linear transformations

A 2×2 matrix acts on a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}.$$

This transforms the plane – scaling, rotation, or reflection.

Example

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ – a scaling by 2.

2 Practice

2.1 State how a matrix acts on a vector. [1]

2.2 Compute $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$. [1]

2.3 Compute $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. [2]

3 Exam-style questions

3.1 The identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ maps a vector to [1]

☐ **A** the zero vector
B itself
C its reverse
D twice itself

3.2 $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ applied to a vector gives a [1]

☐ **A** rotation
B scaling by 3
C reflection
D shift

3.3 $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

(a) Compute $A \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. [1]

(b) Compute $A \begin{bmatrix} 3 \\ 0 \end{bmatrix}$. [1]

(c) Describe the effect. [1]