3.15 Rates of Change in Polar Functions

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS average rate of change

Objective

Build the skills to answer exam questions on **rates of change in polar functions**.

You must be able to:

- compute the **average rate of change** of r with respect to θ
- interpret whether the curve spirals in or out

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Rate of change of r

The average rate of change of r over $[\theta_1, \theta_2]$ is

$$\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}.$$

If r **increases** as θ increases, the curve spirals **outward**; if r decreases, it spirals inward.

Example

$r(\theta) = 3\theta$: $r(0) = 0$, $r(2) = 6$; average rate $= \frac{6-0}{2-0} = 3$.

2 Practice

2.1 State the average rate of change of r over $[\theta_1, \theta_2]$. [1]

2.2 State what it means if r increases as θ increases. [1]

2.3 For $r(\theta) = 2\theta$, find $r(0)$ and $r(\pi)$. [2]

3 Exam-style questions

3.1 The average rate of change of r on $[a, b]$ is [1]

- **A** $r(b) \cdot r(a)$
- **B** $\dfrac{r(b) - r(a)}{b - a}$
- **C** $r(b) + r(a)$
- **D** $b - a$

3.2 If r increases as θ increases, the curve [1]

- **A** spirals inward
- **B** spirals outward
- **C** stays a circle
- **D** is a straight line

3.3 $r(\theta) = 3\theta$.

(a) Find $r(0)$. [1]

(b) Find $r(2)$. [1]

(c) Find the average rate of change on $[0, 2]$. [1]