1.7 Rational Functions and End Behavior

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS horizontal asymptote • rational function • quotient • degree • end behavior

Objective

Build the skills to answer exam questions on **rational functions and end behavior**.

You must be able to:

- describe a **rational function** as a quotient of polynomials
- find its end behavior from the ratio of the **leading terms**
- identify a **horizontal asymptote** where one exists

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Rational functions

A **rational function** $r(x) = \dfrac{p(x)}{q(x)}$ is a quotient of two polynomials.

End behavior

For large $|x|$ each polynomial is dominated by its leading term, so the end behavior is the ratio of the leading terms:

- degree(top) $<$ degree(bottom) $\to$ horizontal asymptote $y = 0$;
- degrees **equal** $\to y = \dfrac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) $>$ degree(bottom) $\to$ **no** horizontal asymptote.

2 Practice

2.1 State what determines the end behavior of a rational function. [1]

2.2 For $r(x) = \dfrac{2x}{x^2+1}$, state the horizontal asymptote. [1]

2.3 For $r(x) = \dfrac{3x^2}{x^2+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 The end behavior of a rational function is found from the ratio of the [1]

- ☐ **A** constant terms
- **B** leading terms
- **C** zeros
- **D** holes

3.2 If the numerator degree is less than the denominator degree, the horizontal asymptote is [1]

- ☐ **A** $y = 1$
- **B** $y = 0$
- **C** none
- **D** $y = x$

3.3 $r(x) = \dfrac{4x^2+1}{2x^2-3}$.

(a) State the horizontal asymptote. [2]

(b) State whether the degrees are equal. [1]