

4.13 Matrices as Functions

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS function • matrix • vector

Objective

Build the skills to answer exam questions on **matrices as functions**.

You must be able to:

- view a matrix transformation $T(\vec{v}) = A\vec{v}$ as a **function** on vectors
- relate composition of transformations to the matrix product

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Matrices as functions

A matrix transformation $T(\vec{v}) = A\vec{v}$ is a function taking an input vector to an output vector. Applying two transformations in sequence corresponds to **multiplying** their matrices: applying A then B is $B(A\vec{v}) = (BA)\vec{v}$.

■ Example

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ swaps components: } T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

2 Practice

2.1 State what the input and output of a matrix transformation are. [1]

2.2 State what composing two matrix transformations corresponds to. [1]

2.3 If $T(\vec{v}) = A\vec{v}$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find $T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$. [2]

3 Exam-style questions

3.1 A matrix transformation $T(\vec{v}) = A\vec{v}$ takes [1]

- ☐ **A** a number to a number
B a vector to a vector
C a matrix to a scalar
D nothing

3.2 Applying transformation A then B equals the single matrix [1]

- ☐ **A** $A + B$
B BA
C $A - B$
D AB only if diagonal

3.3 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Find $A \begin{bmatrix} 2 \\ 5 \end{bmatrix}$. [1]

(b) Find $A \begin{bmatrix} 7 \\ 0 \end{bmatrix}$. [1]

(c) Describe the effect. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the input is a vector and the output is a vector

2.2

- multiplying their matrices

2.3

- $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

3.1 B

- a vector to a vector.

3.2 B

- BA .

3.3

(a)

- $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(b)

- $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$

(c)

- it swaps the two components (a reflection across $y = x$)