3.2 Sine, Cosine, and Tangent

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS radian • unit circle • terminal ray • standard position • arc length • quadrant

Objective

Build the skills to answer exam questions on **sine, cosine and tangent** 、
: radian measure, the unit circle, and the coordinates of a point on a circle.

You must be able to:

- convert between degrees and radians, and use arc length $s = r\theta$
- define $\sin\theta$ and $\cos\theta$ as the coordinates of a point on the unit circle
- find $\sin\theta$, $\cos\theta$ and $\tan\theta$ from a point on the terminal ray
- find the coordinates $(r\cos\theta,\ r\sin\theta)$ of a point on a circle of radius r

1 Key knowledge and worked examples

The unit circle

idea	rule
radian	the angle that cuts off an arc equal to the radius
convert	$180^\circ = \pi$ rad; multiply degrees by $\dfrac{\pi}{180}$
arc length	$s = r\theta$ (θ in radians)
standard position	start on the positive x-axis; positive angles turn anticlockwise
unit circle point	$P = (\cos\theta,\ \sin\theta)$
circle of radius r	$P = (r\cos\theta,\ r\sin\theta)$
point (x, y) on the ray	$r = \sqrt{x^2 + y^2}$; $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, $\tan\theta = \dfrac{y}{x}$
$\tan\theta$	the slope of the terminal ray

Worked example 1: radians. Convert 240°.

1. Known: 240°.
2. Rule: $\times\dfrac{\pi}{180}$.
3. Result: $\dfrac{4\pi}{3}$ **rad**.

Worked example 2: a point on the ray. The terminal ray passes through $(-8, 6)$.

1. Known: $r = \sqrt{64 + 36} = 10$.
2. Rule: $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, $\tan\theta = \dfrac{y}{x}$.
3. Result: $\sin\theta = 0.6$, $\cos\theta = -0.8$, $\tan\theta = -0.75$.

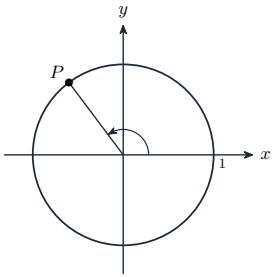
Answer shapes: use radians unless a question says degrees; set your calculator to match.

2 Practice

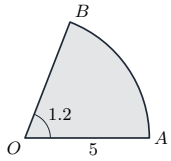
2.1 Convert 150° to radians, and $\dfrac{5\pi}{4}$ radians to degrees. [2]

150° $\dfrac{5\pi}{4}$ rad

2.2 The point $P(-0.6,\ 0.8)$ is on the unit circle at angle θ. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [3]



2.3 A circle has radius 5 cm. Find the length of the arc cut off by an angle of 1.2 radians at the center. [2]



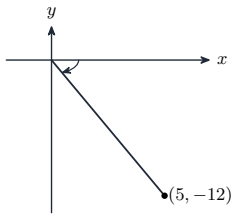
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3 Exam-style questions

3.1 The terminal ray of an angle θ in standard position passes through $(5, -12)$.

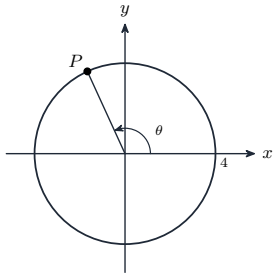
(a) Find the distance from the origin to $(5, -12)$. [1]

(b) Find $\sin\theta$, $\cos\theta$ and $\tan\theta$. [3]



3.2 A point moves anticlockwise round a circle of radius 4 centered at the origin. It starts at $(4, 0)$ and turns through an angle θ.

(a) Find its coordinates when $\theta = 2$ radians, to 3 decimal places. [2]



(b) State the quadrant of the point, and the sign of $\tan 2$. [2]

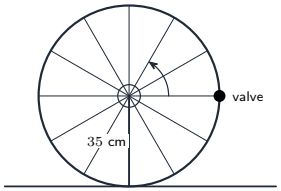
3.3 A bicycle wheel has radius 35 cm. A valve on the rim starts level with the center, on the right, and the wheel turns anticlockwise in the picture. Use the center as the origin.

(a) Find the angle, in radians, that the wheel turns when the valve travels 70 cm along the rim. [2]

(b) Find the coordinates of the valve after this turn. [2]

(c) The center is 35 cm above the ground. Find the valve's height then. [1]

(d) Later the valve is at $(-21, -28)$. Find $\sin\theta$, $\cos\theta$ and $\tan\theta$ for its angle. [3]



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Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

$\frac{\mathbf{5}\pi}{\mathbf{6}}$; $\mathbf{225}^\circ$

2.2

$\sin\theta = \mathbf{0.8}$; $\cos\theta = -\mathbf{0.6}$; $\tan\theta = \dfrac{0.8}{-0.6} = -\frac{\mathbf{4}}{\mathbf{3}}$

2.3

$s = r\theta = 5(1.2) = \mathbf{6\ cm}$

3.1

(a) $\sqrt{25+144} = \mathbf{13}$

(b) $\sin\theta = -\frac{\mathbf{12}}{\mathbf{13}}$; $\cos\theta = \frac{\mathbf{5}}{\mathbf{13}}$; $\tan\theta = -\frac{\mathbf{12}}{\mathbf{5}}$

3.2

(a) $(4\cos 2,\ 4\sin 2) \approx (-\mathbf{1.665},\ \mathbf{3.637})$

(b) **quadrant II**; $\tan 2 = \dfrac{\sin 2}{\cos 2}$ is **negative**

3.3

(a) $\theta = \dfrac{s}{r} = \dfrac{70}{35} = \mathbf{2\ rad}$

(b) $(35\cos 2,\ 35\sin 2) \approx (-\mathbf{14.6},\ \mathbf{31.8})$

(c) $35 + 31.8 \approx \mathbf{66.8\ cm}$

(d) $r = \sqrt{441+784} = 35$; $\sin\theta = -\mathbf{0.8}$; $\cos\theta = -\mathbf{0.6}$; $\tan\theta = \frac{\mathbf{4}}{\mathbf{3}}$

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