3.12 Equivalent Representations of Trigonometric Functions

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS pythagorean identity

Objective

Build the skills to answer exam questions on **equivalent representations of trigonometric functions**.

You must be able to:

- apply the **Pythagorean identity** $\sin^2 x + \cos^2 x = 1$
- use the even/odd symmetry of sine and cosine

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Key identities

- **Pythagorean**: $\sin^2 x + \cos^2 x = 1$, so $\cos^2 x = 1 - \sin^2 x$;
- **even/odd**: $\cos(-x) = \cos x$ (even), $\sin(-x) = -\sin x$ (odd).

Example

If $\sin x = \frac{3}{5}$, then $\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$, so $\cos x = \pm\frac{4}{5}$.

2 Practice

2.1 State the Pythagorean identity. [1]

2.2 Rewrite $1 - \cos^2 x$. [1]

2.3 State $\sin(-x)$ in terms of $\sin x$. [2]

3 Exam-style questions

3.1 The Pythagorean identity is [1]

- ☐ **A** $\sin x + \cos x = 1$
- **B** $\sin^2 x + \cos^2 x = 1$
- **C** $\sin^2 x - \cos^2 x = 1$
- **D** $\sin x \cdot \cos x = 1$

3.2 $\cos(-x)$ equals [1]

- ☐ **A** $-\cos x$
- **B** $\cos x$
- **C** $\sin x$
- **D** $-\sin x$

3.3 Given $\sin x = \frac{3}{5}$.

(a) State $\sin^2 x$. [1]

(b) Use the identity to find $\cos^2 x$. [1]

(c) State a possible value of $\cos x$. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\sin^2 x + \cos^2 x = 1$

2.2

- $\sin^2 x$

2.3

- $\sin(-x) = -\sin x$

3.1 B

- $\sin^2 x + \cos^2 x = 1$.

3.2 B

- $\cos x$.

3.3

(a)

- $\frac{9}{25}$

(b)

- $1 - \frac{9}{25} = \frac{16}{25}$

(c)

- $\frac{4}{5}$ (or $-\frac{4}{5}$)