

1.6 Polynomial Functions and End Behavior

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS end behavior • limit notation • function
• polynomial • leading term

Objective

Build the skills to answer exam questions on **polynomial functions and end behavior**.

You must be able to:

- describe the **end behavior** of a polynomial from its leading term
- use **limit notation** such as $\lim_{x \rightarrow \infty} p(x) = \infty$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ End behavior

The **end behavior** of a polynomial is set by its **leading term** $a_n x^n$:

- even** n , $a_n > 0$: both ends up.
- odd** n , $a_n > 0$: left end down, right end up.

■ Limit notation

”Output increases without bound as x increases” is written $\lim_{x \rightarrow \infty} p(x) = \infty$. For $p(x) = x^3$: $\lim_{x \rightarrow \infty} p(x) = \infty$ and $\lim_{x \rightarrow -\infty} p(x) = -\infty$.

2 Practice

2.1 State what determines a polynomial’s end behavior. [1]

2.2 For $p(x) = x^3$, state the end behavior as $x \rightarrow \infty$ and as $x \rightarrow -\infty$. [2]

2.3 Write the limit notation for ”output increases without bound as x increases”. [1]

3 Exam-style questions

3.1 The end behavior of a polynomial is determined by its [1]

- ☐ **A** constant term
B leading term
C middle term
D zero

3.2 For $p(x) = -2x^4$, as $x \rightarrow \infty$ the output [1]

- ☐ **A** $\rightarrow \infty$
B $\rightarrow -\infty$
C $\rightarrow 0$
D oscillates

3.3 $p(x) = x^3$.

(a) State $\lim_{x \rightarrow \infty} p(x)$. [1]

(b) State $\lim_{x \rightarrow -\infty} p(x)$. [1]

(c) State whether both ends go the same way. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- its leading term (degree and leading coefficient)

2.2

- as $x \rightarrow \infty$, $p(x) \rightarrow \infty$; as $x \rightarrow -\infty$, $p(x) \rightarrow -\infty$

2.3

- $\lim_{x \rightarrow \infty} p(x) = \infty$

3.1 B

- leading term.

3.2 B

- an even degree with a negative leading coefficient sends both ends to $-\infty$

3.3

(a)

- ∞

(b)

- $-\infty$

(c)

- no (opposite directions)