

Exercise walk-through

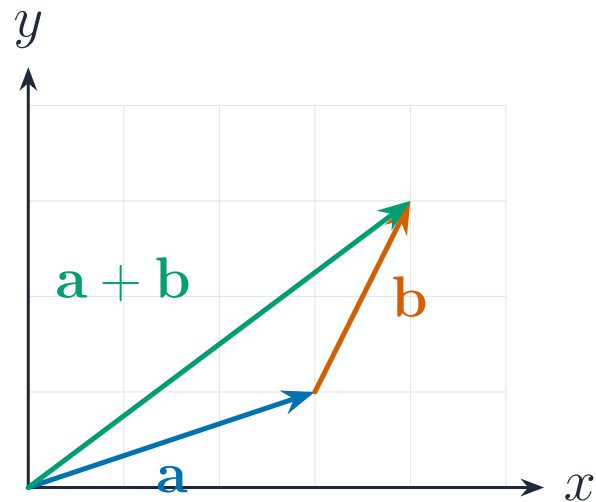
4.7

Parametrization of Implicitly Defined Functions

AP Precalculus

What we will cover

- 1 Parametric functions
- 2 Planar motion & rates
- 3 Circles lines & conics
- 4 Implicit curves
- 5 Vectors
- 6 Matrices & determinants
- 7 Linear transformations

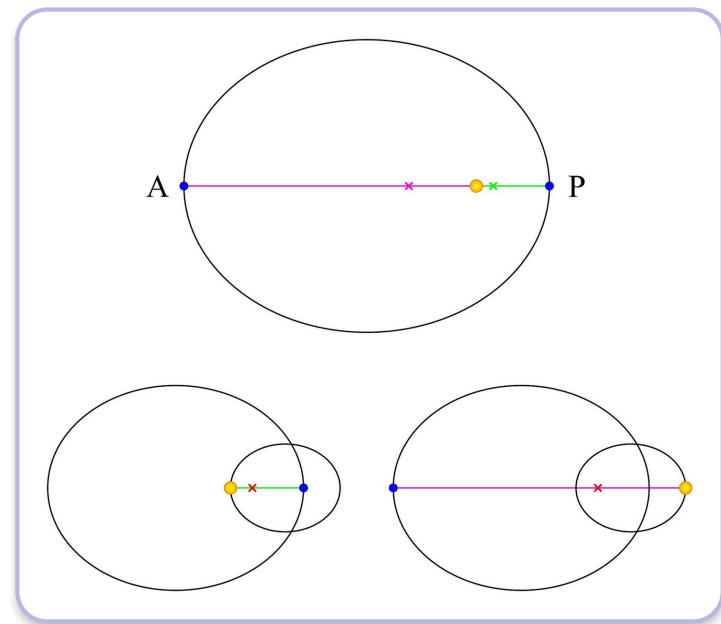


Parametric functions

A **parametric function** gives x and y each as functions of a **parameter** t :

$$x = x(t), \quad y = y(t).$$

The curve traced **need not be a function of x** – it may loop or cross itself.



Elliptical orbit

Implicit curves and conic sections

An **implicitly defined** curve is an equation in x and y – not solved for y . The **conic sections** :

- circle $x^2 + y^2 = r^2$
- ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- parabola $y = ax^2$

To **parametrize** an implicit curve, find $x(t)$ and $y(t)$ that satisfy the equation for every t – for a circle, $\cos^2 t + \sin^2 t = 1$ does it **automatically**.

Exercises 2.1·2.3

2.1 Parametrize $x^2 + y^2 = 16$. [2]

2.2 State the identity used to parametrize a circle. [1]

2.3 Verify that $x = 2 \cos t$, $y = 2 \sin t$ satisfies $x^2 + y^2 = 4$. [1]

Exercises 2.1·2.3

2.1 Parametrize $x^2 + y^2 = 16$. [2]

2.1

- $x = 4 \cos t, y = 4 \sin t$
-

2.2 State the identity used to parametrize a circle. [1]

2.2

- $\cos^2 t + \sin^2 t = 1$
-

2.3 Verify that $x = 2 \cos t, y = 2 \sin t$ satisfies $x^2 + y^2 = 4$. [1]

2.3

- $x^2 + y^2 = 4 \cos^2 t + 4 \sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$

Exercises 3.1·3.2

3.1 The circle $x^2 + y^2 = 25$ parametrizes as

[1]

☐

A $x = 5t, y = 5t$

B $x = 5 \cos t, y = 5 \sin t$

C $x = t^2, y = t^2$

D $x = 25t, y = 25t$

3.2 Parametrizing a circle uses the identity

[1]

☐

A $\sin t + \cos t = 1$

B $\cos^2 t + \sin^2 t = 1$

C $\tan t = \sin t / \cos t$

D $\sec t = 1 / \cos t$

Exercises 3.1·3.2

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3.1 B

- $x = 5 \cos t, y = 5 \sin t.$

3.2 Parametrizing a circle uses the identity

[1]

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A $\sin t + \cos t = 1$

B $\cos^2 t + \sin^2 t = 1$

C $\tan t = \sin t / \cos t$

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3.2 B

- $\cos^2 t + \sin^2 t = 1.$

Exercise 3.3 (a)

3.3 $x^2 + y^2 = 36$.

(a) State r .

[1]

Exercise 3.3 (a)

3.3 $x^2 + y^2 = 36$.

(a) State r .

[1]

(a)

- 6

Exercise 3.3 (b)

3.3 $x^2 + y^2 = 36$.

(b) Write $x(t)$.

[1]

Exercise 3.3 (b)

3.3 $x^2 + y^2 = 36$.

(b) Write $x(t)$.

[1]

(b)

- $x = 6 \cos t$

Exercise 3.3 (c)

3.3 $x^2 + y^2 = 36$.

(c) Write $y(t)$.

[1]

Exercise 3.3 (c)

3.3 $x^2 + y^2 = 36$.

(c) Write $y(t)$.

[1]

(c)

- $y = 6 \sin t$