

Exercise walk-through

4.2

Parametric Functions Modeling Planar Motion

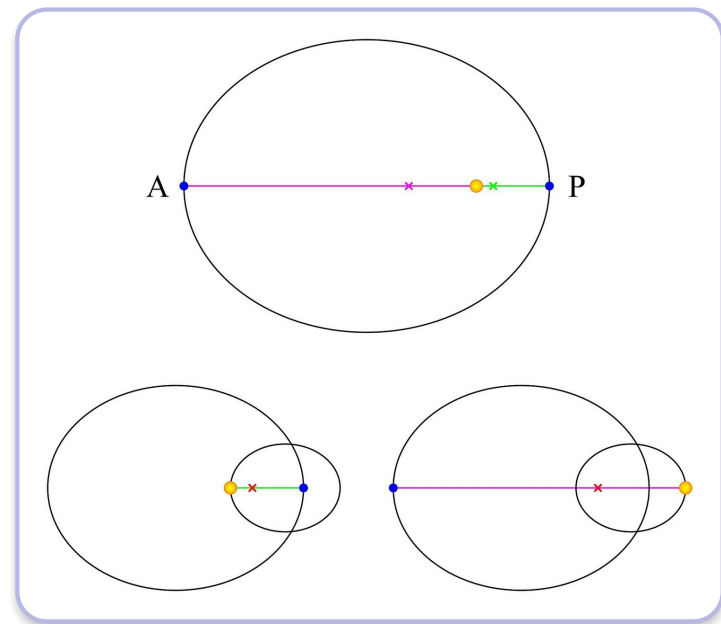
AP Precalculus

Parametric functions

A **parametric function** gives x and y each as functions of a **parameter** t :

$$x = x(t), \quad y = y(t).$$

The curve traced **need not be a function of x** – it may loop or cross itself.



Elliptical orbit

Parametrically Defined Circles and Lines

A **circle** of radius R centered at (h, k) is $x = h + R\cos t$, $y = k + R\sin t$.

A **line** through (x_0, y_0) with direction (a, b) is $x = x_0 + at$, $y = y_0 + bt$.

Parametrising an implicit curve

what it means

rewriting a curve as $(x(t), y(t))$ – a position at each value of t

why bother

it is easier to graph, and it turns the curve into **motion** along a path

the circle

$x = r \cos t$, $y = r \sin t$ traces $x^2 + y^2 = r^2$ once as t runs 0 to 2π

the ellipse

$x = a \cos t$, $y = b \sin t$ – the same idea with different radii

the extra information

a parametrisation also fixes the **direction** and **speed** of travel, which the implicit equation does not

One curve has many parametrisations. Choosing one adds information – where you start, which way you go, how fast – so say what yours does.

Planar motion and rates of change

As t advances, $(x(t), y(t))$ traces the path of a moving point – so t carries **direction** and **timing**, which a plain curve cannot show.

Read each rate **separately**: $x(t)$ increasing moves right, $y(t)$ decreasing moves down.



Vector coaster

Implicit curves and conic sections

An **implicitly defined** curve is an equation in x and y – not solved for y . The **conic sections** :

- circle $x^2 + y^2 = r^2$
- ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- parabola $y = ax^2$

To **parametrize** an implicit curve, find $x(t)$ and $y(t)$ that satisfy the equation for every t – for a circle, $\cos^2 t + \sin^2 t = 1$ does it **automatically**.

Exercises 2.1·2.3

2.1 State what $(x(t), y(t))$ represents in a motion problem. [1]

2.2 State how you find the direction of motion. [1]

2.3 For $x = t$, $y = 3t$, find the positions at $t = 0$ and $t = 1$. [2]

Exercises 2.1·2.3

2.1 State what $(x(t), y(t))$ represents in a motion problem. [1]

2.1

- the object's position in the plane at time t
-

2.2 State how you find the direction of motion. [1]

2.2

- trace the curve in the direction of increasing t
-

2.3 For $x = t$, $y = 3t$, find the positions at $t = 0$ and $t = 1$. [2]

2.3

- $t = 0$: $(0, 0)$; $t = 1$: $(1, 3)$

Exercises 3.1·3.2

3.1 Parametric equations of motion give an object's

[1]

☐

A speed only

B position at each time

C mass

D energy

3.2 The direction of motion is found by

[1]

☐

A increasing t

B decreasing x

C setting $t = 0$

D the amplitude

Exercises 3.1·3.2

3.1 Parametric equations of motion give an object's

[1]

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A speed only

B position at each time

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D energy

3.1 B

- position at each time.

3.2 The direction of motion is found by

[1]

☐

A increasing t

B decreasing x

C setting $t = 0$

D the amplitude

3.2 A

- increasing t .

Exercise 3.3 (a)

3.3 An object has $x = t$, $y = 4 - t$.

(a) State the position at $t = 0$.

[1]

Exercise 3.3 (a)

3.3 An object has $x = t$, $y = 4 - t$.

(a) State the position at $t = 0$.

[1]

(a)

- $(0, 4)$

Exercise 3.3 (b)

3.3 An object has $x = t$, $y = 4 - t$.

(b) State the position at $t = 4$.

[1]

Exercise 3.3 (b)

3.3 An object has $x = t$, $y = 4 - t$.

(b) State the position at $t = 4$.

[1]

(b)

- $(4, 0)$

Exercise 3.3 (c)

3.3 An object has $x = t$, $y = 4 - t$.

(c) Describe the horizontal motion.

[1]

Exercise 3.3 (c)

3.3 An object has $x = t$, $y = 4 - t$.

(c) Describe the horizontal motion.

[1]

(c)

- it moves to the right, since $x = t$ increases with t