

Exercise walk-through

4.11

# The Inverse and Determinant of a Matrix

---

AP Precalculus

# Matrix arithmetic

**a matrix**

a rectangular array of numbers

**addition**

entry by entry, so the two must be the **same size**

**multiplication**

combine **rows of the first with columns of the second**;  $A$ 's column count must equal  $B$ 's row count

**not commutative**

$AB \neq BA$  in general – and often only one of the two is even defined

**the identity**

$I$  leaves any compatible matrix unchanged:  $AI = IA = A$

**the inverse**

$A^{-1}$  exists only when  $\det A \neq 0$ , and then  $AA^{-1} = I$

As a transformation,  $\det A$  is the **area scale factor**. A zero determinant collapses the plane onto a line, which is exactly why the inverse cannot exist.

# A matrix is a function, and it models a system in stages

**the claim**

a matrix maps input vectors to output vectors, so it **is** a function on the plane

**composition**

applying one transformation then another is **matrix multiplication** – and the order matters

**the inverse**

$A^{-1}$  undoes  $A$ , and exists exactly when  $\det A \neq 0$

**modelling in stages**

matrices model systems that **step forward in stages** – populations moving between states

**a transition matrix**

each entry is the proportion moving from one state to another in one step

 **$n$  steps ahead**

multiply by  $A$   $n$  times:  $\mathbf{v}_n = A^n \mathbf{v}_0$

Because composition is multiplication, “do this transformation  $n$  times” becomes “raise the matrix to the  $n$ th power” – which is what makes the stage model tractable.

## Exam tips

- A **parametric** function gives  $x(t)$  and  $y(t)$  separately – track the direction of motion as  $t$  increases.
- A **vector** has magnitude and direction; add component-by-component and find magnitude with  $\sqrt{a^2 + b^2}$ .
- The **determinant** of a  $2 \times 2$  matrix is  $ad - bc$  (mind the minus sign); it scales area under the transformation.
- A  $2 \times 2$  matrix transforms the plane (rotate, reflect, stretch); an inverse exists only when the determinant is non-zero.
- Note this unit is not tested on the AP Precalculus Exam (Units 1–3 only), but it underpins later courses.

## Exercises 2.1-2.3

---

**2.1** State the determinant of  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . [1]

---

**2.2** State when a matrix has an inverse. [1]

---

**2.3** Find the determinant of  $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ . [2]

## Exercises 2.1-2.3

---

**2.1** State the determinant of  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . [1]

**2.1**

- $ad - bc$
- 

**2.2** State when a matrix has an inverse. [1]

**2.2**

- when its determinant is non-zero
- 

**2.3** Find the determinant of  $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ . [2]

**2.3**

- $3(4) - 2(1) = 12 - 2 = 10$

## Exercises 3.1-3.2

---

3.1  $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  equals

[1]

☐

A  $ad + bc$

B  $ad - bc$

C  $ab - cd$

D  $a + d$

☐

3.2 A  $2 \times 2$  matrix has an inverse if and only if

[1]

A it is square

B  $\det \neq 0$

C it is diagonal

D all entries are positive

## Exercises 3.1·3.2

---

3.1  $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  equals

[1]

☐

A  $ad + bc$

B  $ad - bc$

C  $ab - cd$

D  $a + d$

3.1 B

- $ad - bc$ .

---

3.2 A  $2 \times 2$  matrix has an inverse if and only if

[1]

☐

A it is square

B  $\det \neq 0$

C it is diagonal

D all entries are positive

3.2 B

- $\det \neq 0$ .

## Exercise 3.3 (a)

---

$$\mathbf{3.3} \quad A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$$

(a) Find  $\det(A)$ .

[1]

## Exercise 3.3 (a)

---

**3.3**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$

(a) Find  $\det(A)$ .

[1]

(a)

- $2(3) - 0 = 6$

## Exercise 3.3 (b)

---

**3.3**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$

(b) State whether  $A$  is invertible.

[1]

## Exercise 3.3 (b)

---

**3.3**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$

(b) State whether  $A$  is invertible.

[1]

(b)

- yes, since  $\det = 6 \neq 0$

## Exercise 3.3 (c)

---

**3.3**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$

(c) Find  $A^{-1}$ .

[1]

## Exercise 3.3 (c)

---

**3.3**  $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$

(c) Find  $A^{-1}$ .

[1]

(c)

•  $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$