

Exercise walk-through

3.15

Rates of Change in Polar Functions

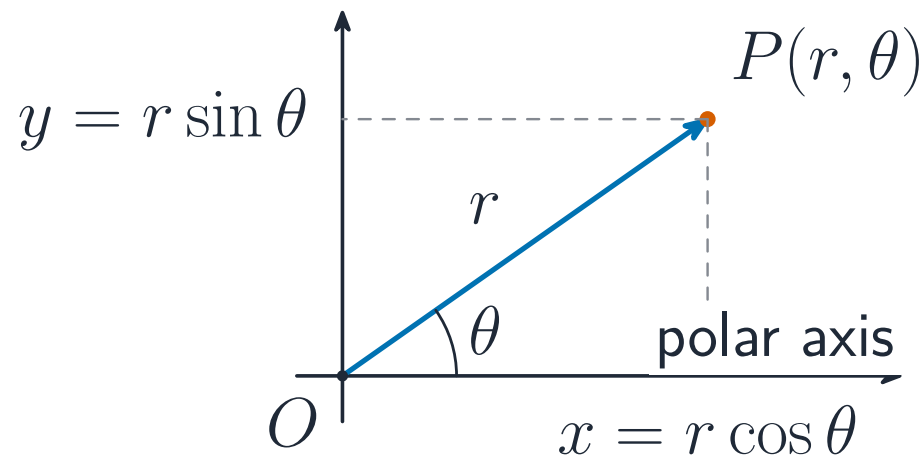
AP Precalculus

Polar coordinates

Polar coordinates give a point by distance r and angle θ :

$$x = r \cos \theta, \quad y = r \sin \theta,$$

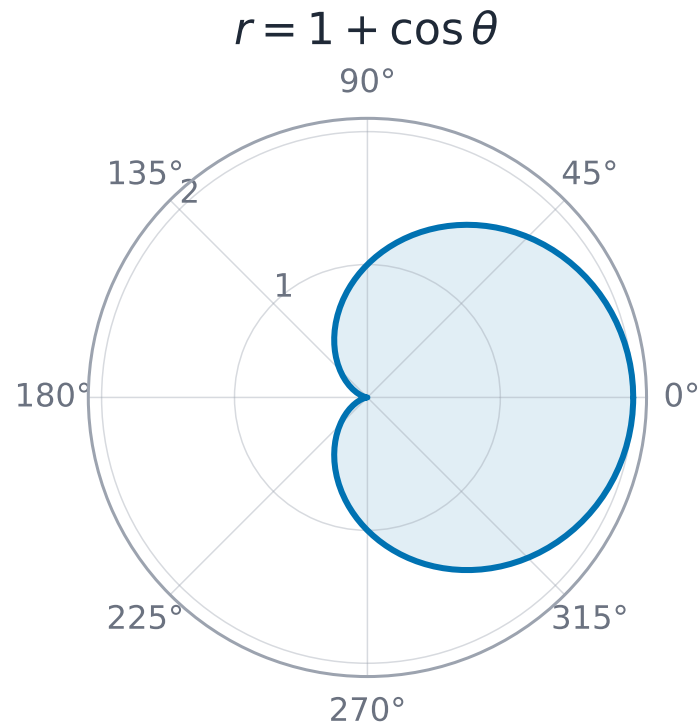
$$r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}.$$



Polar graphs and their rates of change

A polar function $r = f(\theta)$ sweeps a curve as θ turns.

r increasing means the curve moves away from the origin; r decreasing, toward it. A negative r plots opposite the angle.



Reading a polar graph

what it is

the graph of $r = f(\theta)$ is all points (r, θ) as θ sweeps around

restricting θ

draws only **part** of the curve – so the domain is part of the answer

 r increasing

the curve moves **away** from the origin as θ grows

 r decreasing

it moves toward the origin – and a **negative r** plots in the opposite direction

the extremes

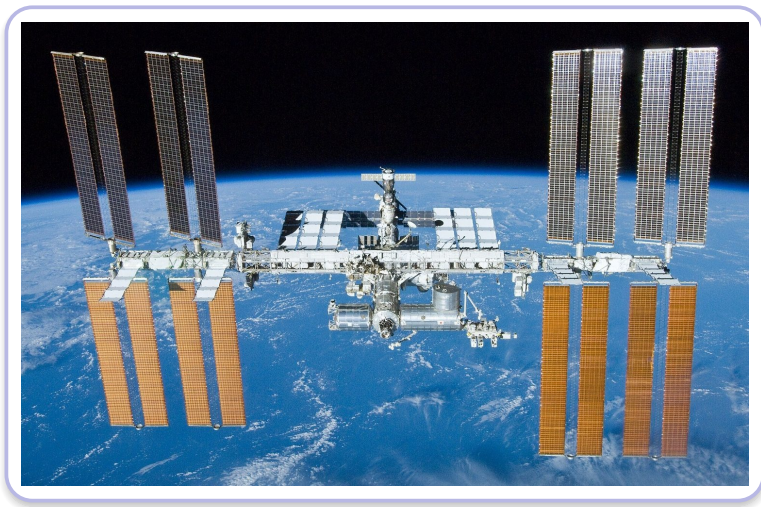
where r switches between increasing and decreasing, the distance from the origin reaches a relative extreme

average rate

$\frac{\Delta r}{\Delta \theta}$ – how fast the distance changes per radian turned

Track **distance from the origin** against **angle**, separately. Trying to read a polar graph as if it were Cartesian is what makes it confusing.

Periodic Phenomena, continued



The ISS in orbit: trigonometric and polar functions describe periodic and circular motion

Exam tips

- Use radians and the **unit circle**: $\cos \theta$ and $\sin \theta$ are the coordinates, always between -1 and 1 .
- For a sinusoid $a \sin(b(\theta + c)) + d$: $|a|$ is the amplitude, $\frac{2\pi}{|b|}$ the period, d the midline, and the phase shift is $-c$ ($a + c$ shifts left).
- Model periodic data by reading period, amplitude, and midline from the max and min.
- **Inverse trig** functions need a restricted domain and return an angle; trig equations have infinitely many solutions (add the period).
- Convert polar $\leftrightarrow$ rectangular with $x = r \cos \theta$, $y = r \sin \theta$.

Exercises 2.1-2.3

2.1 State the average rate of change of r over $[\theta_1, \theta_2]$. [1]

2.2 State what it means if r increases as θ increases. [1]

2.3 For $r(\theta) = 2\theta$, find $r(0)$ and $r(\pi)$. [2]

Exercises 2.1·2.3

2.1 State the average rate of change of r over $[\theta_1, \theta_2]$. [1]

2.1

- $\frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}$

2.2 State what it means if r increases as θ increases. [1]

2.2

- the curve spirals outward (moves farther from the origin)

2.3 For $r(\theta) = 2\theta$, find $r(0)$ and $r(\pi)$. [2]

2.3

- $r(0) = 0$; $r(\pi) = 2\pi$

Exercises 3.1·3.2

3.1 The average rate of change of r on $[a, b]$ is

[1]

☐

- A $r(b) \cdot r(a)$
- B $\frac{r(b) - r(a)}{b - a}$
- C $r(b) + r(a)$
- D $b - a$

3.2 If r increases as θ increases, the curve

[1]

☐

- A spirals inward
- B spirals outward
- C stays a circle
- D is a straight line

Exercises 3.1·3.2

3.1 The average rate of change of r on $[a, b]$ is

[1]

- ☐ **A** $r(b) \cdot r(a)$
B $\frac{r(b) - r(a)}{b - a}$
C $r(b) + r(a)$
D $b - a$

3.1 B

- $\frac{r(b) - r(a)}{b - a}.$

3.2 If r increases as θ increases, the curve

[1]

- ☐ **A** spirals inward
B spirals outward
C stays a circle
D is a straight line

3.2 B

- spirals outward.

Exercise 3.3 (a)

3.3 $r(\theta) = 3\theta$.

(a) Find $r(0)$.

[1]

Exercise 3.3 (a)

3.3 $r(\theta) = 3\theta$.

(a) Find $r(0)$.

[1]

(a)

- 0

Exercise 3.3 (b)

3.3 $r(\theta) = 3\theta$.

(b) Find $r(2)$.

[1]

Exercise 3.3 (b)

3.3 $r(\theta) = 3\theta$.

(b) Find $r(2)$.

[1]

(b)

- 6

Exercise 3.3 (c)

3.3 $r(\theta) = 3\theta$.

(c) Find the average rate of change on $[0, 2]$.

[1]

Exercise 3.3 (c)

3.3 $r(\theta) = 3\theta$.

(c) Find the average rate of change on $[0, 2]$.

[1]

(c)

- $\frac{6-0}{2-0} = 3$