

Exercise walk-through

3.12

Equivalent Representations of Trigonometric Functions

AP Precalculus

What we will cover

- 1 Periodic phenomena
- 2 Sine cosine & tangent
- 3 Sinusoidal functions
- 4 Transformations & modelling
- 5 Inverse trig & equations
- 6 Identities
- 7 Polar functions

The identities the exam expects

Pythagorean identity

$\sin^2 \theta + \cos^2 \theta = 1$, and its two derived forms with $\tan$ and $\cot$

sum identities

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$; $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

double-angle

$\sin 2\theta = 2 \sin \theta \cos \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta$

arcsine

and friends

inverse trig functions need a **restricted domain** to be functions at all

the special triangles

the **equilateral triangle**
60°; the square halved gives 45°

halved gives 30°

Every one of these lets you rewrite an expression into a form you can solve. Identities are **tools for rewriting**, not facts to recite.

Exam tips

- Use radians and the **unit circle**: $\cos \theta$ and $\sin \theta$ are the coordinates, always between -1 and 1 .
- For a sinusoid $a \sin(b(\theta + c)) + d$: $|a|$ is the amplitude, $\frac{2\pi}{|b|}$ the period, d the midline, and the phase shift is $-c$ ($a + c$ shifts left).
- Model periodic data by reading period, amplitude, and midline from the max and min.
- **Inverse trig** functions need a restricted domain and return an angle; trig equations have infinitely many solutions (add the period).
- Convert polar $\leftrightarrow$ rectangular with $x = r \cos \theta$, $y = r \sin \theta$.

Exercises 2.1·2.3

2.1 State the Pythagorean identity.

[1]

2.2 Rewrite $1 - \cos^2 x$.

[1]

2.3 State $\sin(-x)$ in terms of $\sin x$.

[2]

Exercises 2.1·2.3

2.1 State the Pythagorean identity.

[1]

2.1

- $\sin^2 x + \cos^2 x = 1$

2.2 Rewrite $1 - \cos^2 x$.

[1]

2.2

- $\sin^2 x$

2.3 State $\sin(-x)$ in terms of $\sin x$.

[2]

2.3

- $\sin(-x) = -\sin x$

Exercises 3.1·3.2

3.1 The Pythagorean identity is

[1]

☐

A $\sin x + \cos x = 1$

B $\sin^2 x + \cos^2 x = 1$

C $\sin^2 x - \cos^2 x = 1$

D $\sin x \cdot \cos x = 1$

3.2 $\cos(-x)$ equals

[1]

☐

A $-\cos x$

B $\cos x$

C $\sin x$

D $-\sin x$

Exercises 3.1·3.2

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3.1 B

- $\sin^2 x + \cos^2 x = 1.$

3.2 $\cos(-x)$ equals

[1]

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A $-\cos x$

B $\cos x$

C $\sin x$

D $-\sin x$

3.2 B

- $\cos x.$

Exercise 3.3 (a)

3.3 Given $\sin x = \frac{3}{5}$.

(a) State $\sin^2 x$.

[1]

Exercise 3.3 (a)

3.3 Given $\sin x = \frac{3}{5}$.

(a) State $\sin^2 x$.

[1]

(a)

- $\frac{9}{25}$

Exercise 3.3 (b)

3.3 Given $\sin x = \frac{3}{5}$.

(b) Use the identity to find $\cos^2 x$.

[1]

Exercise 3.3 (b)

3.3 Given $\sin x = \frac{3}{5}$.

(b) Use the identity to find $\cos^2 x$.

[1]

(b)

- $1 - \frac{9}{25} = \frac{16}{25}$

Exercise 3.3 (c)

3.3 Given $\sin x = \frac{3}{5}$.

(c) State a possible value of $\cos x$.

[1]

Exercise 3.3 (c)

3.3 Given $\sin x = \frac{3}{5}$.

(c) State a possible value of $\cos x$.

[1]

(c)

- $\frac{4}{5}$ (or $-\frac{4}{5}$)