

Exercise walk-through

3.11

The Secant, Cosecant, and Cotangent Functions

AP Precalculus

The identities the exam expects

Pythagorean identity

$\sin^2 \theta + \cos^2 \theta = 1$, and its two derived forms with $\tan$ and $\cot$

sum identities

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$; $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

double-angle

$\sin 2\theta = 2 \sin \theta \cos \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta$

arcsine

and friends

inverse trig functions need a **restricted domain** to be functions at all

the special triangles

the **equilateral triangle**

60°; the square halved gives 45°

halved gives 30°

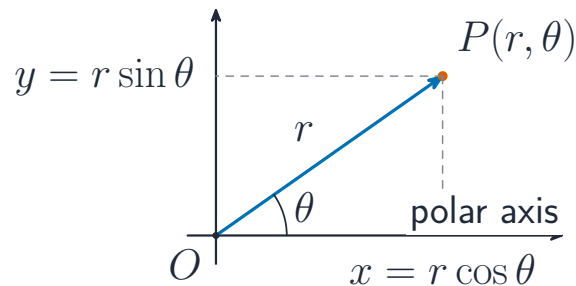
Every one of these lets you rewrite an expression into a form you can solve. Identities are **tools for rewriting**, not facts to recite.

Exam tips

- Use radians and the **unit circle**: $\cos \theta$ and $\sin \theta$ are the coordinates, always between -1 and 1 .
- For a sinusoid $a \sin(b(\theta + c)) + d$: $|a|$ is the amplitude, $\frac{2\pi}{|b|}$ the period, d the midline, and the phase shift is $-c$ ($a + c$ shifts left).
- Model periodic data by reading period, amplitude, and midline from the max and min.
- **Inverse trig** functions need a restricted domain and return an angle; trig equations have infinitely many solutions (add the period).
- Convert polar $\leftrightarrow$ rectangular with $x = r \cos \theta$, $y = r \sin \theta$.

Check yourself

- 1 Give the amplitude, period and midline of $3 \sin(2\theta) + 1$.
- 2 Solve $2 \sin \theta = 1$ on $[0, 2\pi)$.
- 3 Why does $\arcsin$ need a restricted domain?
- 4 Convert: what are x and y in terms of r and θ ?



Answers 1. $3, \pi, y = 1$ 2. $\frac{\pi}{6}, \frac{5\pi}{6}$ 3. sine is not one-to-one 4. $x = r \cos \theta, y = r \sin \theta$

Exercises 2.1-2.3

2.1 State $\sec x$ in terms of $\cos x$.

[1]

2.2 State $\csc x$ in terms of $\sin x$.

[1]

2.3 Evaluate $\sec 0$ and $\csc \frac{\pi}{2}$.

[2]

Exercises 2.1-2.3

2.1 State $\sec x$ in terms of $\cos x$.

[1]

2.1

- $\sec x = 1/\cos x$
-

2.2 State $\csc x$ in terms of $\sin x$.

[1]

2.2

- $\csc x = 1/\sin x$
-

2.3 Evaluate $\sec 0$ and $\csc \frac{\pi}{2}$.

[2]

2.3

- $\sec 0 = 1$; $\csc \frac{\pi}{2} = 1$

Exercises 3.1-3.2

3.1 $\sec x$ equals

[1]

☐

A $1/\sin x$

B $1/\cos x$

C $1/\tan x$

D $\cos x$

3.2 $\cot x$ equals

[1]

☐

A $\sin x/\cos x$

B $\cos x/\sin x$

C $1/\sin x$

D $\tan x$

Exercises 3.1-3.2

3.1 $\sec x$ equals

[1]

☐

A $1/\sin x$

B $1/\cos x$

C $1/\tan x$

D $\cos x$

3.1 B

- $1/\cos x$.

3.2 $\cot x$ equals

[1]

☐

A $\sin x/\cos x$

B $\cos x/\sin x$

C $1/\sin x$

D $\tan x$

3.2 B

- $\cos x/\sin x$.

Exercise 3.3 (a)

3.3 Answer each.

(a) Evaluate $\sec 0$.

[1]

Exercise 3.3 (a)

3.3 Answer each.

(a) Evaluate $\sec 0$.

[1]

(a)

- 1

Exercise 3.3 (b)

3.3 Answer each.

(b) Evaluate $\csc \frac{\pi}{2}$.

[1]

Exercise 3.3 (b)

3.3 Answer each.

(b) Evaluate $\csc \frac{\pi}{2}$.

[1]

(b)

- 1

Exercise 3.3 (c)

3.3 Answer each.

(c) State where $\csc x$ is undefined.

[1]

Exercise 3.3 (c)

3.3 Answer each.

(c) State where $\csc x$ is undefined.

[1]

(c)

- where $\sin x = 0$ (i.e. $x = n\pi$)