

Exercise walk-through

3.10

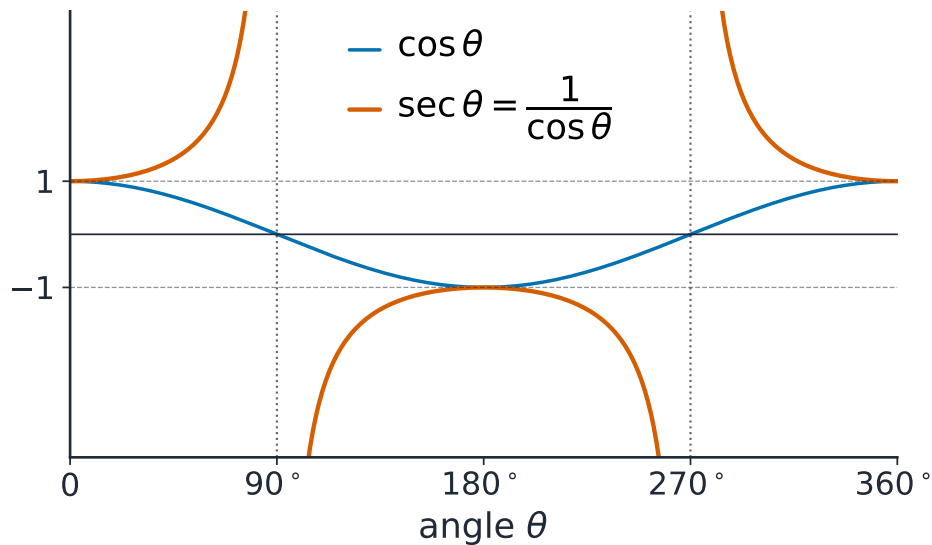
Trigonometric Equations and Inequalities

AP Precalculus

Tangent, and the reciprocal functions

$$\sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}.$$

$\tan \theta$ has period π (not 2π) and a **vertical asymptote** wherever $\cos \theta = 0$.



Inverse trigonometric functions

Sine, cosine and tangent are **not** one-to-one, so each inverse needs a **restricted domain**.

An inverse trig function **returns an angle** – and only the one in its restricted range.

arcsin: range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

arccos: range $[0, \pi]$

arctan: range $(-\frac{\pi}{2}, \frac{\pi}{2})$

Worked example – a trigonometric equation

Solve $2 \sin \theta = 1$ **on** $[0, 2\pi)$

$\sin \theta = \frac{1}{2}$. On the unit circle sine is positive in the **first and second** quadrants:

$$\theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6}.$$

Over all reals, add $2\pi k$ to each.

A trig equation has **infinitely many** solutions – the calculator gives you **one**. Add the period, and find the **second** quadrant solution yourself.

Equivalent representations – the identities

Pythagorean: $\sin^2 \theta + \cos^2 \theta = 1$

Sum: $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

Double angle: $\sin 2\theta = 2 \sin \theta \cos \theta$

An **identity** is true for **every** θ – use one to rewrite an expression into a form you can solve.

The identities the exam expects

Pythagorean identity

$\sin^2 \theta + \cos^2 \theta = 1$, and its two derived forms with $\tan$ and $\cot$

sum identities

$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$; $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

double-angle

$\sin 2\theta = 2 \sin \theta \cos \theta$; $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 1 - 2 \sin^2 \theta$

arcsine

and friends

inverse trig functions need a **restricted domain** to be functions at all

the special triangles

the **equilateral triangle**

60°; the square halved gives 45°

halved gives 30°

Every one of these lets you rewrite an expression into a form you can solve. Identities are **tools for rewriting**, not facts to recite.

Exercises 2.1-2.3

2.1 Solve $\sin x = 0$ on $[0, 2\pi)$. [1]

2.2 State why a trig equation has infinitely many solutions. [1]

2.3 Solve $\cos x = 1$ on $[0, 2\pi)$. [2]

Exercises 2.1·2.3

2.1 Solve $\sin x = 0$ on $[0, 2\pi)$. [1]

2.1

- $x = 0$ and $x = \pi$
-

2.2 State why a trig equation has infinitely many solutions. [1]

2.2

- sine and cosine are periodic, so each solution repeats every 2π
-

2.3 Solve $\cos x = 1$ on $[0, 2\pi)$. [2]

2.3

- $\cos x = 1$ at $x = 0$ only on $[0, 2\pi)$

Exercises 3.1·3.2

3.1 The equation $\sin x = \frac{1}{2}$ has

[1]

☐

A one solution in total

B two solutions in $[0, 2\pi)$

C no solution

D no solution in $[0, 2\pi)$

☐

3.2 Trig equations have infinitely many solutions because sine and cosine are

[1]

A linear

B periodic

C polynomial

D rational

Exercises 3.1·3.2

3.1 The equation $\sin x = \frac{1}{2}$ has

[1]

☐

- A one solution in total
- B two solutions in $[0, 2\pi)$
- C no solution
- D no solution in $[0, 2\pi)$

3.1 B

- two solutions in $[0, 2\pi)$.

3.2 Trig equations have infinitely many solutions because sine and cosine are

[1]

☐

- A linear
- B periodic
- C polynomial
- D rational

3.2 B

- periodic.

Exercise 3.3 (a)

3.3 Solve $\sin x = 1$.

(a) State the solution on $[0, 2\pi)$.

[1]

Exercise 3.3 (a)

3.3 Solve $\sin x = 1$.

(a) State the solution on $[0, 2\pi)$.

[1]

(a)

- $x = \frac{\pi}{2}$

Exercise 3.3 (b)

3.3 Solve $\sin x = 1$.

(b) State the general solution (all x).

[1]

Exercise 3.3 (b)

3.3 Solve $\sin x = 1$.

(b) State the general solution (all x).

[1]

(b)

- $x = \frac{\pi}{2} + 2\pi n$

Exercise 3.3 (c)

3.3 Solve $\sin x = 1$.

(c) State the amount added each time.

[1]

Exercise 3.3 (c)

3.3 Solve $\sin x = 1$.

(c) State the amount added each time.

[1]

(c)

- 2π