Exercise walk-through

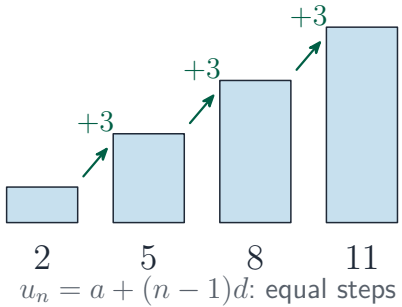
2.9
Logarithmic Expressions

AP Precalculus

Arithmetic and geometric sequences

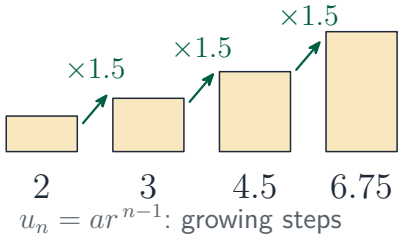
A **sequence** maps whole numbers to reals: its graph is discrete points.
Arithmetic adds the same amount; geometric multiplies by the same ratio.

arithmetic: add d each step



$u_n = a + (n-1)d$: equal steps

geometric: multiply by r each step



$u_n = ar^{n-1}$: growing steps

arithmetic	: $a_n = a_0 + dn$ with **common difference**	d
geometric	: $g_n = g_0\, r^n$ with **common ratio**	r

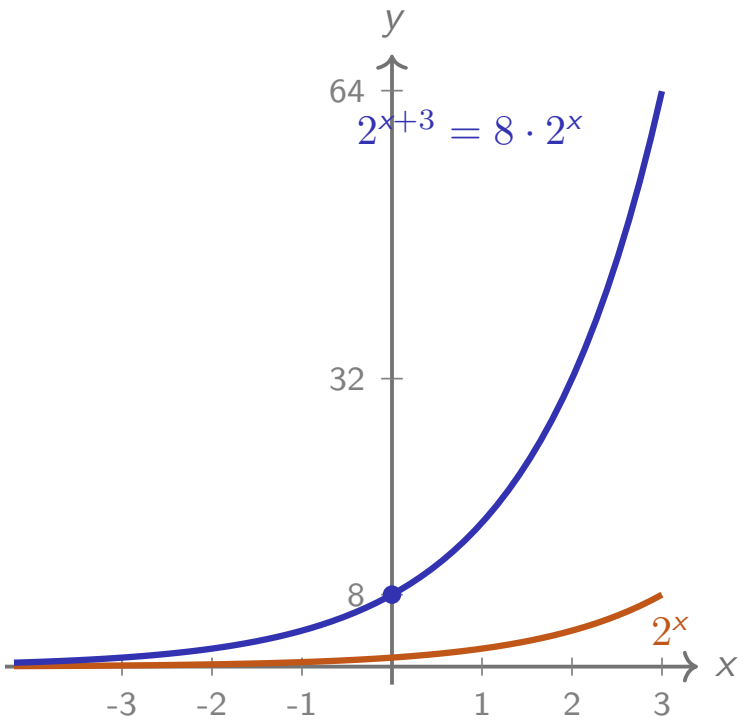
A shift is a stretch

Rewrite $f(x) = 2^{x+3}$

Product rule: $2^{x+3} = 2^3 \cdot 2^x = 8 \cdot 2^x$.

Shifting 2^x left 3 gives the same graph as stretching it vertically by 8.

> Any horizontal shift of b^x is a vertical stretch by b^k.



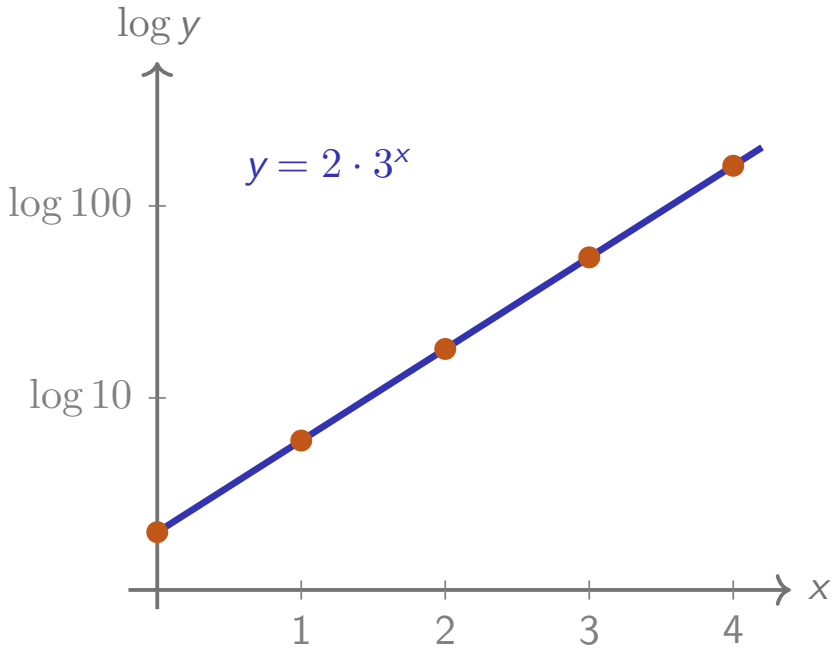
Semi-log plots

On a logarithmic scale, each unit is a multiplicative step $(10^0, 10^1, 10^2, \dots)$.

$$y = ab^x \;\Rightarrow\; \log y = \log a + x \log b$$

slope $\log b$, intercept $\log a$

An exponential becomes a straight line.



Exercises 2.1–2.3

2.1 Rewrite $\log_b(y) = x$ in exponential form. [1]

2.2 Evaluate $\log_2(8)$. [1]

2.3 Evaluate $\log_{10}(1000)$. [2]

Exercises 2.1–2.3

2.1 Rewrite $\log_b(y) = x$ in exponential form. [1]

2.1

- $b^x = y$

2.2 Evaluate $\log_2(8)$. [1]

2.2

- 3

2.3 Evaluate $\log_{10}(1000)$. [2]

2.3

- $\log_{10}(1000) = 3$ since $10^3 = 1000$

Exercises 3.1·3.2

3.1 $\log_b(y) = x$ is equivalent to [1]

- [ ] **A** $b^y = x$
- [ ] **B** $b^x = y$
- [ ] **C** $x^b = y$
- [ ] **D** $y^x = b$

3.2 $\log_3(9)$ equals [1]

- [ ] **A** 2
- [ ] **B** 3
- [ ] **C** 9
- [ ] **D** 27

Exercises 3.1·3.2

3.1 $\log_b(y) = x$ is equivalent to [1]

- [ ] **A** $b^y = x$
- [ ] **B** $b^x = y$
- [ ] **C** $x^b = y$
- [ ] **D** $y^x = b$

3.1 B

- $b^x = y$.

3.2 $\log_3(9)$ equals [1]

- [ ] **A** 2
- [ ] **B** 3
- [ ] **C** 9
- [ ] **D** 27

3.2 A

- 2.

Exercise 3.3 (a)

3.3 Evaluate each.

(a) $\log_5(25)$. [1]

Exercise 3.3 (a)

3.3 Evaluate each.

(a) $\log_5(25)$. [1]

(a)

- 2

Exercise 3.3 (b)

3.3 Evaluate each.

(b) $\log_{10}(100)$. [1]

Exercise 3.3 (b)

3.3 Evaluate each.

(b) $\log_{10}(100)$. [1]

(b)

- 2

Exercise 3.3 (c)

3.3 Evaluate each.

(c) $\log_2(1)$. [1]

Exercise 3.3 (c)

3.3 Evaluate each.

(c) $\log_2(1)$. [1]

(c)

- 0

Exercise 3.4

3.4 (a) Evaluate $\log_3(1/27)$ and check it in exponential form. [2]

(b) Explain why $\log_3(0)$ has no real value, although a negative logarithm can exist. [2]

Exercise 3.4

3.4 (a) Evaluate $\log_3(1/27)$ and check it in exponential form. [2]

(b) Explain why $\log_3(0)$ has no real value, although a negative logarithm can exist. [2]

3.4

(a) $3^{-3} = 1/27$, so $\log_3(1/27) = -3$.

(b) 3^x is positive for every real x, so it never equals zero; negative exponents instead give positive fractions such as $1/27$.