

Exercise walk-through

2.7

Composition of Functions

AP Precalculus

Composition of functions

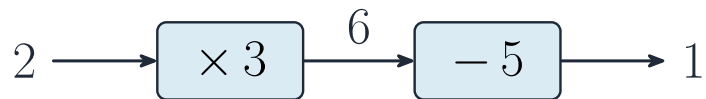
The **composite** $(f \circ g)(x) = f(g(x))$ feeds g 's output into f .

Composition is **not commutative**: $f(g(x))$ and $g(f(x))$ usually differ.

To build it, **substitute** $g(x)$ for every x in f . The **identity** $f(x) = x$ changes nothing.

Composition of functions, in detail

$f(x) = 3x - 5$: do $\times 3$, then -5



$f^{-1}(x) = \frac{x + 5}{3}$



reverse the order, undo each step

the inverse runs it backwards: $f(2) = 1$, so $f^{-1}(1) = 2$

The **composite**

$(f \circ g)(x) = f(g(x))$ feeds g 's output into f .

Composing, decomposing, and inverting

composite function

$(f \circ g)(x) = f(g(x))$ – inner function first, and the order matters

the domain

x must be allowed in g , and $g(x)$ must be allowed in f

decomposing

splitting a function into simpler pieces – an additive shift is composing with $x + k$, a dilation is composing with bx

finding an inverse

swap x and y in $y = f(x)$, then solve for y

the check

$f(f^{-1}(x)) = x$: composing a function with its inverse gives the **identity function**

the restriction

context, or a failed horizontal-line test, may limit where the inverse applies

Decomposition is why transformations and composition are the same topic: every shift and stretch is a composition with a simple function.

Inverse functions

f is **invertible** where each output comes from a **unique** input. Then $f(a) = b \Rightarrow f^{-1}(b) = a$, and

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

Domain and range **swap**; the graph reflects over $y = x$.

swap x and y in $y = f(x)$



solve for y

You may **restrict the domain** to force invertibility.

Exercises 2.1·2.3

2.1 State what $(f \circ g)(x)$ means.

[1]

2.2 For $f(x) = x + 2$ and $g(x) = 3x$, find $(f \circ g)(x)$.

[2]

2.3 For the same functions, find $(g \circ f)(x)$.

[1]

Exercises 2.1·2.3

2.1 State what $(f \circ g)(x)$ means.

[1]

2.1

- apply g first, then f : $f(g(x))$

2.2 For $f(x) = x + 2$ and $g(x) = 3x$, find $(f \circ g)(x)$.

[2]

2.2

- $(f \circ g)(x) = 3x + 2$

2.3 For the same functions, find $(g \circ f)(x)$.

[1]

2.3

- $(g \circ f)(x) = (x + 2) \cdot 3 = 3x + 6$

Exercises 3.1·3.2

3.1 $(f \circ g)(x)$ means

[1]

☐

A $f(x) \cdot g(x)$

B $f(g(x))$

C $g(f(x))$

D $f(x) + g(x)$

3.2 In a composition, you apply the

[1]

☐

A outer function first

B inner function first

C both at once

D neither

Exercises 3.1·3.2

3.1 $(f \circ g)(x)$ means

[1]

☐

A $f(x) \cdot g(x)$

B $f(g(x))$

C $g(f(x))$

D $f(x) + g(x)$

3.1 B

- $f(g(x))$.

3.2 In a composition, you apply the

[1]

☐

A outer function first

B inner function first

C both at once

D neither

3.2 B

- inner function first.

Exercise 3.3 (a)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$.

[1]

Exercise 3.3 (a)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$.

[1]

(a)

- $2(x - 1) = 2x - 2$

Exercise 3.3 (b)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(b) Find $(g \circ f)(x)$.

[1]

Exercise 3.3 (b)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(b) Find $(g \circ f)(x)$.

[1]

(b)

- $2x - 1$

Exercise 3.3 (c)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(c) Evaluate $(f \circ g)(4)$.

[1]

Exercise 3.3 (c)

3.3 $f(x) = 2x$, $g(x) = x - 1$.

(c) Evaluate $(f \circ g)(4)$.

[1]

(c)

- $2(4 - 1) = 6$