

Exercise walk-through

# 2.5

## Exponential Function Context and Data Modeling

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AP Precalculus

# What makes a function exponential

## the defining property

outputs are **proportional** over equal-length input intervals  
– a constant *ratio*, where a linear function has a constant difference

## the consequence

an exponential is always increasing or always decreasing, and always concave up or always concave down

## exponential growth

$b > 1$ : the value multiplies each step

## exponential decay

$0 < b < 1$ : it multiplies by less than one, approaching zero without reaching it

## in data

sometimes a **constant must be added** to the data before the proportional pattern shows – a shifted asymptote hides it

Test data by **dividing** consecutive outputs, not subtracting. A constant ratio is exponential; a constant difference is linear.

# Modelling, and choosing between models

Exponentials model growth by a **constant proportion** over equal intervals.

- build from a ratio + an initial value, or from **two points**
- **exponential regression** fits a data set
- the **natural base**  
 $e \approx 2.718$  is the standard



*Exponential sprint*

# Choosing between competing models

**the problem**

when a rate of change shifts only slightly, linear, quadratic and exponential can all seem to fit

**residual**

actual minus predicted – the gap the model leaves at each point

**residual plot**

a model is **appropriate** when its residual plot shows **no pattern**

**a pattern means**

the model is missing structure the data has – a curve in the residuals means you fitted a line to a curve

**the error**

the gap between predicted and actual; **context** decides whether over- or under-estimating is safer

A high  $R^2$  with a patterned residual plot is still the wrong model. The residual plot is the check that catches what the summary number hides.

## Exercises 2.1-2.3

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**2.1** State the growth factor for 5% growth per year.

[1]

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**2.2** State the decay factor for 20% decay per year.

[1]

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**2.3** Write a model for \$200 growing 10% per year.

[2]

## Exercises 2.1-2.3

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**2.1** State the growth factor for 5% growth per year. [1]

**2.1**

- 1.05
- 

**2.2** State the decay factor for 20% decay per year. [1]

**2.2**

- 0.80
- 

**2.3** Write a model for \$200 growing 10% per year. [2]

**2.3**

- $f(x) = 200 \cdot (1.10)^x$

## Exercises 3.1-3.2

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**3.1** A quantity grows 3% per year. The growth factor  $b$  is

[1]

☐

A 0.03

B 0.97

C 1.03

D 3

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☐

**3.2** A quantity decays 15% per year. The factor  $b$  is

[1]

A 1.15

B 0.85

C 0.15

D  $-0.15$

## Exercises 3.1-3.2

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**3.1** A quantity grows 3% per year. The growth factor  $b$  is

[1]

☐

A 0.03

B 0.97

C 1.03

D 3

**3.1 C**

- 1.03.

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**3.2** A quantity decays 15% per year. The factor  $b$  is

[1]

☐

A 1.15

B 0.85

C 0.15

D  $-0.15$

**3.2 B**

- 0.85.

## Exercise 3.3 (a)

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**3.3** A colony of 100 bacteria doubles each hour.

(a) State the initial value.

[1]

## Exercise 3.3 (a)

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**3.3** A colony of 100 bacteria doubles each hour.

(a) State the initial value.

[1]

(a)

- 100

## Exercise 3.3 (b)

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**3.3** A colony of 100 bacteria doubles each hour.

(b) State the growth factor.

[1]

## Exercise 3.3 (b)

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**3.3** A colony of 100 bacteria doubles each hour.

(b) State the growth factor.

[1]

(b)

- 2

## Exercise 3.3 (c)

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**3.3** A colony of 100 bacteria doubles each hour.

(c) Write the model  $f(t)$ .

[1]

## Exercise 3.3 (c)

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**3.3** A colony of 100 bacteria doubles each hour.

(c) Write the model  $f(t)$ .

[1]

(c)

- $f(t) = 100 \cdot 2^t$