

Exercise walk-through

2.2

Change in Linear and Exponential Functions

AP Precalculus

What we will cover

- 1 Arithmetic & geometric sequences
- 2 Linear vs exponential change
- 3 Exponential functions
- 4 Composition & inverses
- 5 Logarithms
- 6 Solving equations
- 7 Modelling & semi-log plots

Choosing between competing models

the problem

when a rate of change shifts only slightly, linear, quadratic and exponential can all seem to fit

residual

actual minus predicted – the gap the model leaves at each point

residual plot

a model is **appropriate** when its residual plot shows **no pattern**

a pattern means

the model is missing structure the data has – a curve in the residuals means you fitted a line to a curve

the error

the gap between predicted and actual; **context** decides whether over- or under-estimating is safer

A high R^2 with a patterned residual plot is still the wrong model. The residual plot is the check that catches what the summary number hides.

Exam tips

- Distinguish **arithmetic** (add a common difference) from **geometric** (multiply a common ratio) sequences and their linear/exponential function cousins.
- Exponential = repeated **multiplication**, so it eventually outgrows any linear or polynomial model.
- The **logarithm is the inverse** of the exponential ($\log_b c = a \Leftrightarrow b^a = c$); use it to solve $b^x = k$.
- Apply the log laws (product, quotient, power) and the change-of-base formula; the argument of a log must be **positive**.
- On a **semi-log plot** an exponential model becomes a straight line.

Exercises 2.1-2.3

2.1 State how a linear function changes over equal intervals. [1]

2.2 State how an exponential function changes over equal intervals. [1]

2.3 A function multiplies by 3 each step. Is it linear or exponential? [1]

Exercises 2.1·2.3

2.1 State how a linear function changes over equal intervals. [1]

2.1

- by equal differences (it adds a constant amount)
-

2.2 State how an exponential function changes over equal intervals. [1]

2.2

- by equal ratios (it multiplies by a constant factor)
-

2.3 A function multiplies by 3 each step. Is it linear or exponential? [1]

2.3

- exponential

Exercises 3.1-3.2

3.1 A function that adds a constant each unit is

[1]

☐

A exponential

B linear

C quadratic

D rational

3.2 A function that multiplies by a constant each unit is

[1]

☐

A linear

B exponential

C constant

D polynomial

Exercises 3.1-3.2

3.1 A function that adds a constant each unit is

[1]

☐

A exponential

B linear

C quadratic

D rational

3.1 B

- linear.

3.2 A function that multiplies by a constant each unit is

[1]

☐

A linear

B exponential

C constant

D polynomial

3.2 B

- exponential.

Exercise 3.3 (a)

3.3 The values $4, 12, 36, 108, \dots$ are recorded at equal intervals.

(a) Is the change by equal difference or equal ratio?

[1]

Exercise 3.3 (a)

3.3 The values 4, 12, 36, 108, ... are recorded at equal intervals.

(a) Is the change by equal difference or equal ratio?

[1]

(a)

- equal ratio

Exercise 3.3 (b)

3.3 The values $4, 12, 36, 108, \dots$ are recorded at equal intervals.

(b) State the constant.

[1]

Exercise 3.3 (b)

3.3 The values $4, 12, 36, 108, \dots$ are recorded at equal intervals.

(b) State the constant.

[1]

(b)

- 3

Exercise 3.3 (c)

3.3 The values $4, 12, 36, 108, \dots$ are recorded at equal intervals.

(c) State the model type.

[1]

Exercise 3.3 (c)

3.3 The values $4, 12, 36, 108, \dots$ are recorded at equal intervals.

(c) State the model type.

[1]

(c)

- exponential