

Exercise walk-through

2.14

Logarithmic Function Context and Data Modeling

AP Precalculus

What makes a function exponential

the defining property

outputs are **proportional** over equal-length input intervals
– a constant *ratio*, where a linear function has a constant difference

the consequence

an exponential is always increasing or always decreasing, and always concave up or always concave down

exponential growth

$b > 1$: the value multiplies each step

exponential decay

$0 < b < 1$: it multiplies by less than one, approaching zero without reaching it

in data

sometimes a **constant must be added** to the data before the proportional pattern shows – a shifted asymptote hides it

Test data by **dividing** consecutive outputs, not subtracting. A constant ratio is exponential; a constant difference is linear.

Choosing between competing models

the problem

when a rate of change shifts only slightly, linear, quadratic and exponential can all seem to fit

residual

actual minus predicted – the gap the model leaves at each point

residual plot

a model is **appropriate** when its residual plot shows **no pattern**

a pattern means

the model is missing structure the data has – a curve in the residuals means you fitted a line to a curve

the error

the gap between predicted and actual; **context** decides whether over- or under-estimating is safer

A high R^2 with a patterned residual plot is still the wrong model. The residual plot is the check that catches what the summary number hides.

Composing, decomposing, and inverting

composite function

$(f \circ g)(x) = f(g(x))$ – inner function first, and the order matters

the domain

x must be allowed in g , and $g(x)$ must be allowed in f

decomposing

splitting a function into simpler pieces – an additive shift is composing with $x + k$, a dilation is composing with bx

finding an inverse

swap x and y in $y = f(x)$, then solve for y

the check

$f(f^{-1}(x)) = x$: composing a function with its inverse gives the **identity function**

the restriction

context, or a failed horizontal-line test, may limit where the inverse applies

Decomposition is why transformations and composition are the same topic: every shift and stretch is a composition with a simple function.

Exercises 2.1-2.3

2.1 State one real quantity measured on a logarithmic scale.

[1]

2.2 On a base-10 log scale, a change of 1 unit means what factor?

[1]

2.3 State the shape of a logarithmic growth curve.

[1]

Exercises 2.1-2.3

2.1 State one real quantity measured on a logarithmic scale. [1]

2.1

- any of: pH, sound level in decibels, earthquake magnitude (Richter)

2.2 On a base-10 log scale, a change of 1 unit means what factor? [1]

2.2

- a factor of 10

2.3 State the shape of a logarithmic growth curve. [1]

2.3

- rises quickly at first, then levels off (increasing and concave down)

Exercises 3.1-3.2

3.1 The Richter scale for earthquakes is

[1]

☐

A linear

B logarithmic

C exponential

D quadratic

3.2 On a base-10 log scale, +2 units corresponds to a factor of

[1]

☐

A 2

B 10

C 20

D 100

Exercises 3.1-3.2

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3.2 D

- 100.

Exercise 3.3 (a)

3.3 A sound rises from 40 dB to 60 dB (a base-10 scale where $+10 \text{ dB} = \times 10$ intensity).

(a) State the increase in dB.

[1]

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(a)

- 20 dB

Exercise 3.3 (b)

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(b) State how many factors of 10 this is. [1]

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(b)

- 2

Exercise 3.3 (c)

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(c) State the intensity factor.

[1]

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(c) State the intensity factor. [1]

(c)

- $\times 100$