

Exercise walk-through

2.10

Inverses of Exponential Functions

AP Precalculus

Composition of functions

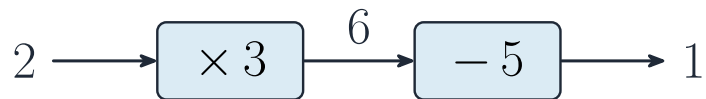
The **composite** $(f \circ g)(x) = f(g(x))$ feeds g 's output into f .

Composition is **not commutative**: $f(g(x))$ and $g(f(x))$ usually differ.

To build it, **substitute** $g(x)$ for every x in f . The **identity** $f(x) = x$ changes nothing.

Composition of functions, in detail

$f(x) = 3x - 5$: do $\times 3$, then -5



$f^{-1}(x) = \frac{x + 5}{3}$



reverse the order, undo each step

the inverse runs it backwards: $f(2) = 1$, so $f^{-1}(1) = 2$

The **composite**

$(f \circ g)(x) = f(g(x))$ feeds g 's output into f .

Composing, decomposing, and inverting

composite function

$(f \circ g)(x) = f(g(x))$ – inner function first, and the order matters

the domain

x must be allowed in g , and $g(x)$ must be allowed in f

decomposing

splitting a function into simpler pieces – an additive shift is composing with $x + k$, a dilation is composing with bx

finding an inverse

swap x and y in $y = f(x)$, then solve for y

the check

$f(f^{-1}(x)) = x$: composing a function with its inverse gives the **identity function**

the restriction

context, or a failed horizontal-line test, may limit where the inverse applies

Decomposition is why transformations and composition are the same topic: every shift and stretch is a composition with a simple function.

Inverse functions

f is **invertible** where each output comes from a **unique** input. Then $f(a) = b \Rightarrow f^{-1}(b) = a$, and

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

Domain and range **swap**; the graph reflects over $y = x$.

swap x and y in $y = f(x)$



solve for y

You may **restrict the domain** to force invertibility.

Exercises 2.1·2.3

2.1 State the inverse of $f(x) = 10^x$.

[1]

2.2 Solve $3^x = 81$.

[2]

2.3 State the inverse of e^x .

[1]

Exercises 2.1·2.3

2.1 State the inverse of $f(x) = 10^x$. [1]

2.1

- $f^{-1}(x) = \log_{10}(x)$
-

2.2 Solve $3^x = 81$. [2]

2.2

- $3^x = 81 = 3^4$, so $x = 4$
-

2.3 State the inverse of e^x . [1]

2.3

- $\ln(x)$ (i.e. $\log_e(x)$)

Exercises 3.1·3.2

3.1 The inverse of $f(x) = b^x$ is

[1]

☐

A b^{-x}

B $\log_b(x)$

C $1/b^x$

D x^b

3.2 To solve $5^x = 125$, take

[1]

☐

A $\log_5$ of both sides

B the square root

C the reciprocal

D the derivative

Exercises 3.1·3.2

3.1 The inverse of $f(x) = b^x$ is

[1]

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B $\log_b(x)$

C $1/b^x$

D x^b

3.1 B

- $\log_b(x)$.

3.2 To solve $5^x = 125$, take

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A $\log_5$ of both sides

B the square root

C the reciprocal

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3.2 A

- $\log_5$ of both sides.

Exercise 3.3 (a)

3.3 Solve $2^x = 32$.

(a) Rewrite 32 as a power of 2.

[1]

Exercise 3.3 (a)

3.3 Solve $2^x = 32$.

(a) Rewrite 32 as a power of 2.

[1]

(a)

- $32 = 2^5$

Exercise 3.3 (b)

3.3 Solve $2^x = 32$.

(b) Equate the exponents.

[1]

Exercise 3.3 (b)

3.3 Solve $2^x = 32$.

(b) Equate the exponents.

[1]

(b)

- $x = 5$

Exercise 3.3 (c)

3.3 Solve $2^x = 32$.

(c) State x .

[1]

Exercise 3.3 (c)

3.3 Solve $2^x = 32$.

(c) State x .

[1]

(c)

- 5