

Exercise walk-through

1.8

Rational Functions and Zeros

AP Precalculus

Rational end behaviour – compare the degrees

top $>$ bottom
follows the **quotient** polynomial;
if linear, a **slant asymptote**

equal degrees
horizontal asymptote
at the **ratio of leading coefficients**

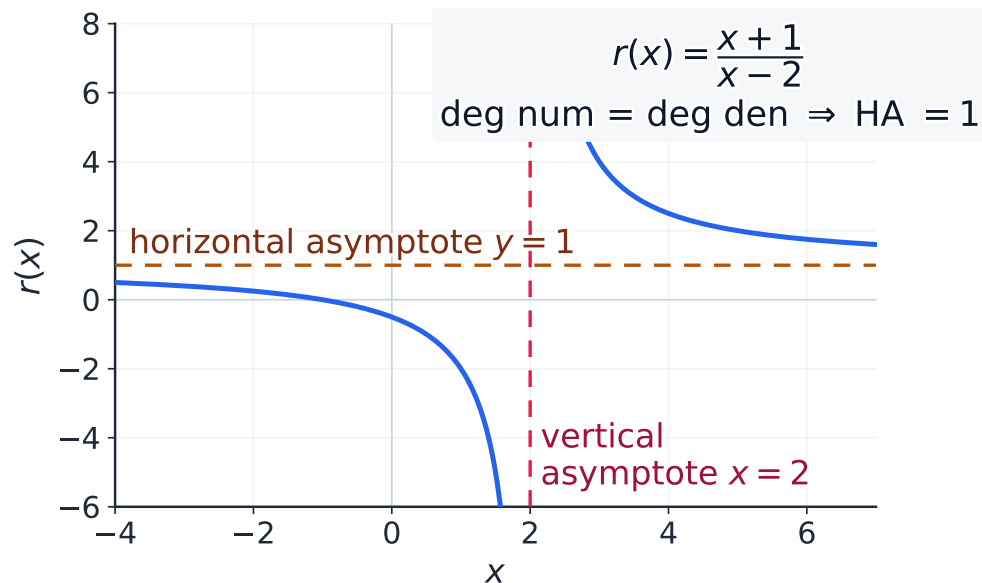
top $<$ bottom
the quotient
 $\rightarrow 0$, so $y = 0$

At a horizontal asymptote $y = b$, the outputs get **and stay** arbitrarily close to b .

Vertical asymptotes and holes

- a **vertical asymptote** at $x = a$: a is a zero of the **denominator** that does **not** cancel
- a **hole** at $x = c$: the factor **cancels** – a single missing point at (c, L)

Check **each side** of an asymptote – the two sides can go opposite ways.



Worked example – describe a rational function

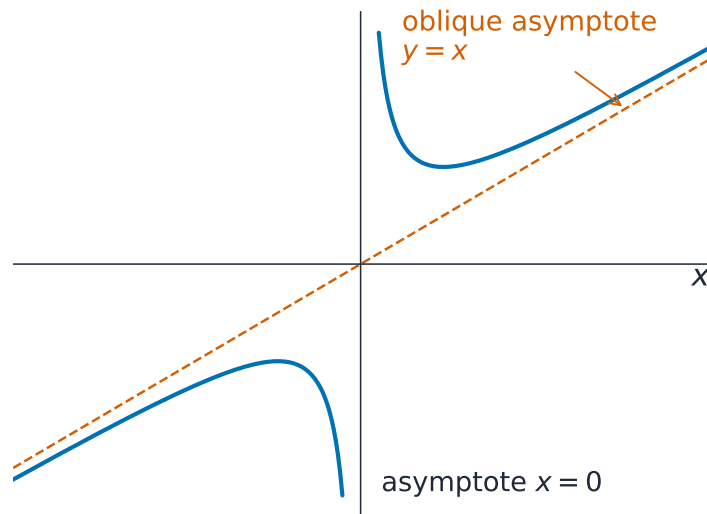
$$r(x) = \frac{2x^2 + 3}{x^2 - 1}$$

Equal degrees, so the horizontal asymptote is

$$y = \frac{2}{1} = 2.$$

$x^2 - 1 = 0$ at $x = \pm 1$, and neither cancels, so there are vertical asymptotes at

$$x = 1 \quad \text{and} \quad x = -1.$$



Top degree **one more** than the bottom: long division gives a **slant** asymptote.

Equivalent forms reveal different features

Factored form

shows real **zeros** – and hence intercepts, holes, vertical asymptotes, domain and range

Standard form

shows the **degree** and **leading term** – and hence end behaviour

Long division

rewrites $f = gq + r$; the **quotient** q is the **slant asymptote** when the degrees differ by one.

The same expression, written to reveal different things

factored form

shows the **zeros** immediately

standard form

shows the **degree** and the leading coefficient, so the end behaviour

vertex form

shows the **vertex** of a quadratic without any calculation

after polynomial long division

shows a rational function's **asymptote** as the quotient, plus a remainder that vanishes

the habit

before calculating, ask which form already *contains* the answer

Rewriting is not busywork; it is choosing the representation in which the question is already answered.

Exercises 2.1·2.3

2.1 State where the real zeros of a rational function come from.

[1]

2.2 For $r(x) = \frac{x-4}{x+1}$, state the zero.

[1]

2.3 State why a numerator zero might **not** be a zero of r .

[1]

Exercises 2.1·2.3

2.1 State where the real zeros of a rational function come from.

[1]

2.1

- from the real zeros of the numerator (that lie in the domain)

2.2 For $r(x) = \frac{x-4}{x+1}$, state the zero.

[1]

2.2

- $x = 4$

2.3 State why a numerator zero might **not** be a zero of r .

[1]

2.3

- if it is also a zero of the denominator, it is outside the domain (a hole, not a zero)

Exercises 3.1-3.2

3.1 The real zeros of a rational function come from the

[1]

☐

A denominator

B numerator

C leading term

D degree

3.2 The function $r(x) = \frac{x-2}{x-2}$

[1]

☐

A has a zero at 2

B has a hole at 2

C has an asymptote at 2

D is undefined everywhere

Exercises 3.1·3.2

3.1 The real zeros of a rational function come from the

[1]

- ☐ **A** denominator
B numerator
C leading term
D degree

3.1 B

- numerator.

3.2 The function $r(x) = \frac{x-2}{x-2}$

[1]

- ☐ **A** has a zero at 2
B has a hole at 2
C has an asymptote at 2
D is undefined everywhere

3.2 B

- has a hole at 2.

Exercise 3.3 (a)

$$\mathbf{3.3} \quad r(x) = \frac{x^2 - 9}{x - 1}.$$

(a) State the zeros of the numerator.

[1]

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[1]

(a)

- $x = 3$ and $x = -3$

Exercise 3.3 (b)

3.3 $r(x) = \frac{x^2 - 9}{x - 1}.$

(b) State the zeros of r .

[1]

Exercise 3.3 (b)

3.3 $r(x) = \frac{x^2 - 9}{x - 1}.$

(b) State the zeros of r .

[1]

(b)

- $x = 3$ and $x = -3$

Exercise 3.3 (c)

$$\mathbf{3.3} \quad r(x) = \frac{x^2 - 9}{x - 1}.$$

(c) State the value excluded from the domain.

[1]

Exercise 3.3 (c)

$$\mathbf{3.3} \quad r(x) = \frac{x^2 - 9}{x - 1}.$$

(c) State the value excluded from the domain.

[1]

(c)

- $x = 1$