

Exercise walk-through

1.5

Polynomial Functions and Complex Zeros

AP Precalculus

Rational end behaviour – compare the degrees

top $>$ bottom
follows the **quotient** polynomial;
if linear, a **slant asymptote**

equal degrees
horizontal asymptote
at the **ratio of leading coefficients**

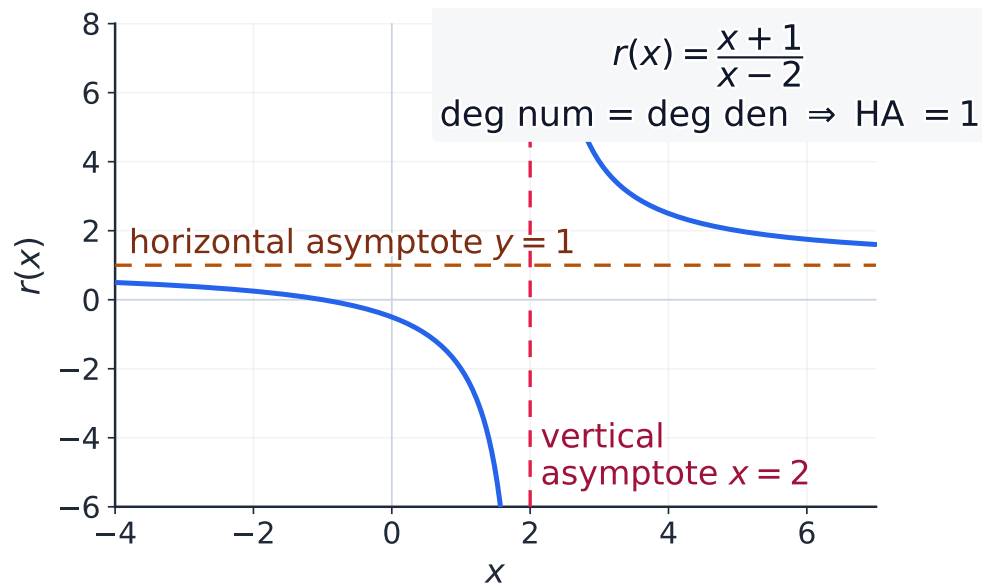
top $<$ bottom
the quotient
 $\rightarrow 0$, so $y = 0$

At a horizontal asymptote $y = b$, the outputs get **and stay** arbitrarily close to b .

Vertical asymptotes and holes

- a **vertical asymptote** at $x = a$: a is a zero of the **denominator** that does **not** cancel
- a **hole** at $x = c$: the factor **cancel**s – a single missing point at (c, L)

Check **each side** of an asymptote – the two sides can go opposite ways.



Worked example – describe a rational function

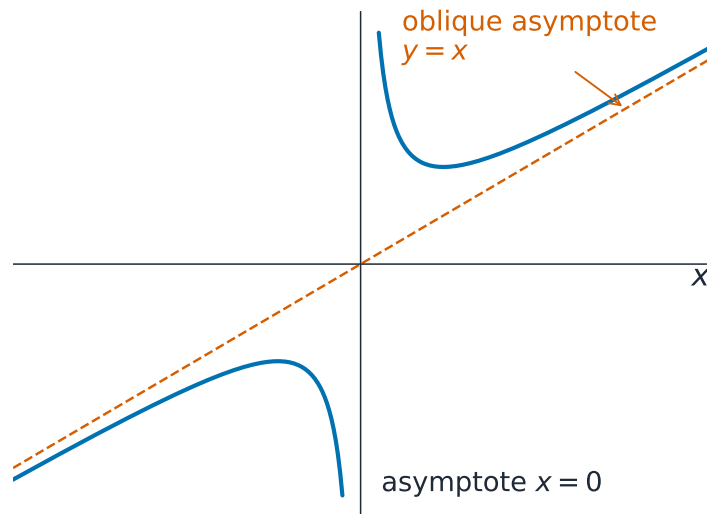
$$r(x) = \frac{2x^2 + 3}{x^2 - 1}$$

Equal degrees, so the horizontal asymptote is

$$y = \frac{2}{1} = 2.$$

$x^2 - 1 = 0$ at $x = \pm 1$, and neither cancels, so there are vertical asymptotes at

$$x = 1 \quad \text{and} \quad x = -1.$$



Top degree **one more** than the bottom: long division gives a **slant** asymptote.

Equivalent forms reveal different features

Factored form

shows real **zeros** – and hence intercepts, holes, vertical asymptotes, domain and range

Standard form

shows the **degree** and **leading term** – and hence end behaviour

Long division

rewrites $f = gq + r$; the **quotient** q is the **slant asymptote** when the degrees differ by one.

The same expression, written to reveal different things

factored form

shows the **zeros** immediately

standard form

shows the **degree** and the leading coefficient, so the end behaviour

vertex form

shows the **vertex** of a quadratic without any calculation

after polynomial long division

shows a rational function's **asymptote** as the quotient, plus a remainder that vanishes

the habit

before calculating, ask which form already *contains* the answer

Rewriting is not busywork; it is choosing the representation in which the question is already answered.

Exercises 2.1·2.3

2.1 State the relationship between a zero a and the factor $(x - a)$. [1]

2.2 State how many complex zeros a degree-4 polynomial has (counting multiplicity). [1]

2.3 For $p(x) = (x - 2)^2(x + 1)$, state the zeros and their multiplicities. [2]

Exercises 2.1·2.3

2.1 State the relationship between a zero a and the factor $(x - a)$. [1]

2.1

- $(x - a)$ is a factor of p if and only if a is a zero of p
-

2.2 State how many complex zeros a degree-4 polynomial has (counting multiplicity). [1]

2.2

- 4
-

2.3 For $p(x) = (x - 2)^2(x + 1)$, state the zeros and their multiplicities. [2]

2.3

- $x = 2$ (multiplicity 2); $x = -1$ (multiplicity 1)

Exercises 3.1-3.2

3.1 If $p(a) = 0$, then $(x - a)$ is a

[1]

☐

A vertical asymptote

B factor of p

C hole

D leading term

3.2 A degree-5 polynomial has _____ complex zeros counting multiplicity.

[1]

☐

A 1

B 3

C 5

D 10

Exercises 3.1-3.2

3.1 If $p(a) = 0$, then $(x - a)$ is a

[1]

☐

A vertical asymptote

B factor of p

C hole

D leading term

3.1 B

- factor of p .

3.2 A degree-5 polynomial has _____ complex zeros counting multiplicity.

[1]

☐

A 1

B 3

C 5

D 10

3.2 C

- 5.

Exercise 3.3 (a)

3.3 $p(x) = (x - 3)^3(x + 2)$.

(a) State the zeros.

[1]

Exercise 3.3 (a)

3.3 $p(x) = (x - 3)^3(x + 2)$.

(a) State the zeros.

[1]

(a)

- $x = 3$ and $x = -2$

Exercise 3.3 (b)

3.3 $p(x) = (x - 3)^3(x + 2)$.

(b) State the multiplicity of $x = 3$.

[1]

Exercise 3.3 (b)

3.3 $p(x) = (x - 3)^3(x + 2)$.

(b) State the multiplicity of $x = 3$.

[1]

(b)

- 3

Exercise 3.3 (c)

3.3 $p(x) = (x - 3)^3(x + 2).$

(c) State the degree.

[1]

Exercise 3.3 (c)

3.3 $p(x) = (x - 3)^3(x + 2)$.

(c) State the degree.

[1]

(c)

- 4