HOLIDAY HOMEWORK 假期作业

AP Precalculus

Exercise sheets and past-paper practice, topic by topic.

每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

Contents 目录

AP PRECALCULUS • HOLIDAY HOMEWORK 假期作业

1 Functions and rates of change – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
function	函数			
domain	定义域			
range	值域			
increasing	递增			
decreasing	递减			
average rate of change	平均变化率			
secant line	割线			
linear	线性			
quadratic	二次			

Recall

You have already worked with functions and graphs:

- A function turns each input into one output, like $f(x) = x^2$.
- You have found the gradient (slope) of a straight line.
- You know a quadratic makes a U-shaped parabola.

This sheet lines up how functions change. Each idea has a worked example.

New ideas

1 Functions, domain, and range

A **function** 函数 gives each input exactly one output. The allowed inputs are the **domain** 定义域; the outputs are the **range** 值域.

2 Increasing and decreasing

A function is **increasing** 递增 where a bigger input gives a bigger output, and **decreasing** 递减 where a bigger input gives a smaller output.

3 Average rate of change

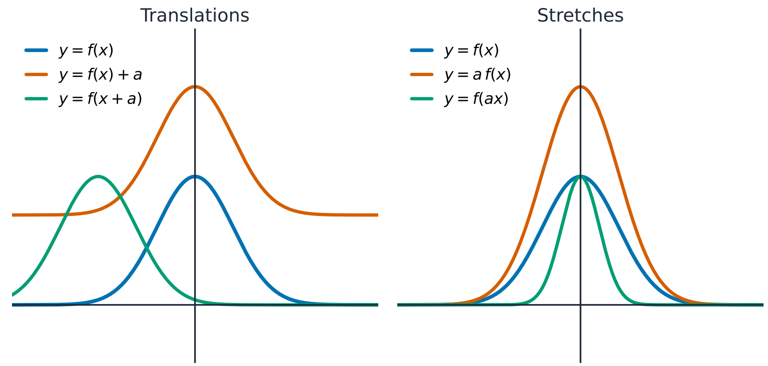
The **average rate of change** 平均变化率 over an interval $[a, b]$ is the change in output over the change in input:

$$\text{avg rate} = \frac{f(b) - f(a)}{b - a}.$$

This is just the gradient of the straight line (the **secant line** 割线) joining the two points.

Worked example. For $f(x) = x^2$ over $[1, 4]$: $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{16 - 1}{3} = 5$.

4 Linear versus quadratic



For a **linear** 线性 function the average rate of change is **constant**; for a **quadratic** 二次 function it **changes** from interval to interval.

Watch out: the average rate of change is the slope of the line between two points – so for a curve it depends on **which** two points you pick. Only a straight line has the same rate everywhere.

Practice

Try these in order. The methods are all above.

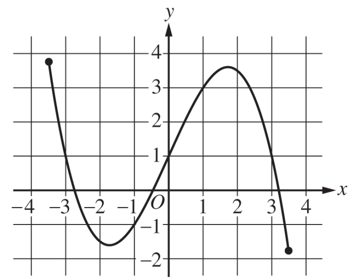
2.1 State what the domain and the range of a function are. [2]

2.2 For $f(x) = 3x + 1$, find the average rate of change over $[0, 4]$. [2]

2.3 For $f(x) = x^2$, find the average rate of change over $[2, 5]$. [3]

2.4 State whether $f(x) = x^2$ is increasing or decreasing for $x > 0$. [1]

2.5 Explain why a linear function has the same average rate of change over every interval. [2]



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

   (A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

   (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

   (B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

   (ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

   (C) (i) Determine if f has an inverse function.

   (ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. A musician released a new song on a streaming service. A streaming service is an online entertainment source that allows users to play music on their computers and mobile devices.

Several months later, the musician began using an app (at time $t = 0$) that counts the total number of plays for the song since its release. A "play" is a single stream of the song on the streaming service. The table gives the total number of plays, in thousands, for selected times t months after the musician began using the app. At $t = 0$, the total number of plays was 25 thousand. At $t = 2$, the total number of plays was 30 thousand. At $t = 4$, the total number of plays was 34 thousand.

Months after the musician began using the app	0	2	4
Total number of plays for the song since its release (thousands)	25	30	34

The total number of plays, in thousands, for the song since its release can be modeled by the function D given by $D(t) = at^2 + bt + c$, where $D(t)$ is the total number of plays, in thousands, for the song since its release, and t is the number of months after the musician began using the app.

A.

i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $D(t)$.

ii. Find the values for a, b, and c as decimal approximations.

B.

i. Use the given data to find the average rate of change of the total number of plays for the song, in thousands per month, from $t = 0$ to $t = 4$ months. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. Show the work that leads to your answer.

iii. Let A_t represent the estimate of the total number of plays for the song, in thousands, using the average rate of change found in part B (i). For $A_{1.5}$ found in part B (ii), it can be shown that $A_{1.5} < D(1.5)$.
Explain why, in general, $A_t < D(t)$ for all t, where $0 < t < 4$. Your explanation should include a reference to the graph of D and its relationship to A_t.

C. The quadratic function model D has exactly one absolute minimum or one absolute maximum. That minimum or maximum can be used to determine a domain restriction for D.

Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

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END OF PART A

1 Polynomials and rational functions – Part 2

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Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
polynomial	多项式			
degree	次数			
zero	零点			
end behavior	末端行为			
rational function	有理函数			
vertical asymptote	垂直渐近线			
horizontal asymptote	水平渐近线			

Recall

In Part A you met functions and their rates of change. Now two important families:

- **Polynomials** like $x^3 - 2x + 1$ – sums of powers of x.
- **Rational functions** – one polynomial divided by another.

Each idea has a worked example before the Practice.

New ideas

1 Polynomials

A **polynomial** 多项式 is a sum of powers of x, like $p(x) = 2x^3 - 5x + 1$. Its highest power is the **degree** 次数, and the term with that power is the **leading term**.

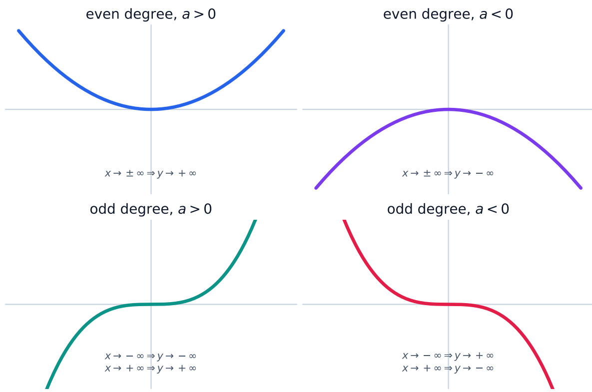
2 Zeros

A **zero** 零点 (or root) of p is an input where $p(x) = 0$. Each real zero gives an **x-intercept** where the graph crosses the x-axis.

Worked example. $p(x) = (x-2)(x+3)$ has zeros at $x = 2$ and $x = -3$, so its graph crosses the x-axis at those two points.

3 End behavior

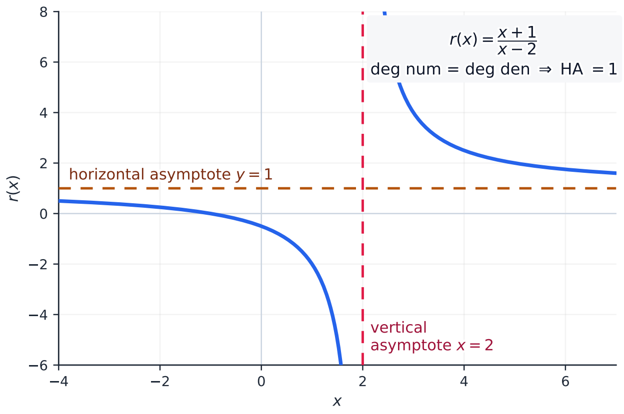
The **end behavior** 末端行为 – where the graph heads as $x \to \pm\infty$ – is decided entirely by the leading term.



for $y = ax^n$: parity of n + sign of a fix both ends as $x \to \pm\infty$

4 Rational functions and asymptotes

A **rational function** 有理函数 is one polynomial over another. A **vertical asymptote** 垂直渐近线 appears where the denominator is zero (and the top is not), and a **horizontal asymptote** 水平渐近线 describes where the graph levels off far out.



Worked example. For $r(x) = \dfrac{1}{x-2}$, the denominator is zero at $x = 2$, so there is a vertical asymptote there; far out the value tends to 0, so $y = 0$ is a horizontal asymptote.

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one – the function shoots off to $\pm\infty$ as it approaches a vertical asymptote.

Practice

Try these in order. The methods are all above.

2.1 State the degree of $p(x) = 4x^3 - 2x^2 + 7$. [1]

2.2 Find the zeros of $p(x) = (x-1)(x+4)$. [2]

2.3 State where the graph of $p(x) = (x-1)(x+4)$ crosses the x-axis. [1]

2.4 For $r(x) = \dfrac{1}{x-3}$, state the vertical asymptote. [1]

2.5 For $r(x) = \dfrac{5}{x+1}$, state the horizontal asymptote as $x \to \pm\infty$. [1]

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

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Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of D.

END OF PART A

2 Sequences and exponential functions – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
sequence	数列			
arithmetic sequence	等差数列			
common difference	公差			
geometric sequence	等比数列			
common ratio	公比			
exponential function	指数函数			
exponential growth	指数增长			
exponential decay	指数衰减			

Recall

You have seen number patterns and powers before:

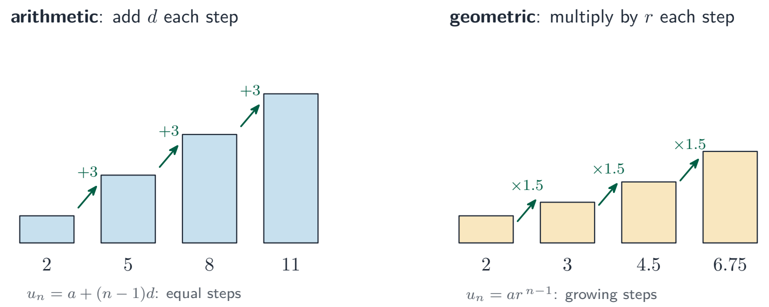
- Sequences like $2, 4, 6, 8, \ldots$ (adding) or $2, 4, 8, 16, \ldots$ (doubling).
- Powers like $2^3 = 8$.
- Some things grow faster and faster – like money earning interest.

This sheet lines up sequences and exponential growth. Each idea has a worked example.

New ideas

1 Two kinds of sequence

A **sequence** 数列 is an ordered list of numbers. Two important types:



- an **arithmetic sequence** 等差数列 **adds** a fixed **common difference** 公差 d each step,
- a **geometric sequence** 等比数列 **multiplies** by a fixed **common ratio** 公比 r each step.

Worked example. Arithmetic with $a_0 = 3$, $d = 5$: the terms are $3, 8, 13, 18, \ldots$ Geometric with $g_0 = 2$, $r = 3$: $2, 6, 18, 54, \ldots$

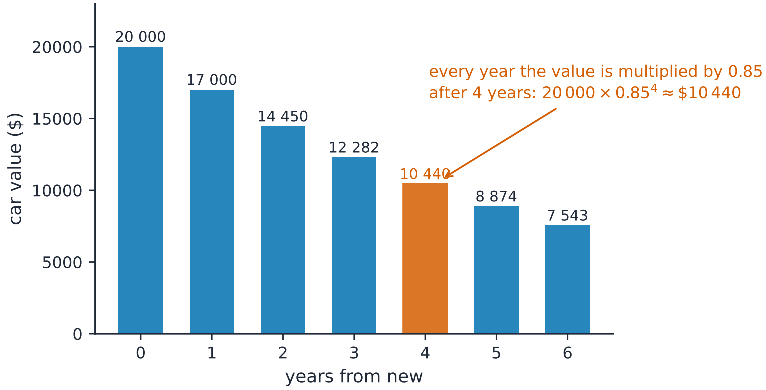
2 Finding a term

For arithmetic, $a_n = a_0 + dn$; for geometric, $g_n = g_0\, r^n$.

Worked example. $a_4 = 3 + 5(4) = 23$; $g_4 = 2 \cdot 3^4 = 162$.

3 Exponential functions

An **exponential function** 指数函数 is $f(x) = ab^x$ – a starting value a multiplied by the **base** b repeatedly.



- $b > 1$ gives **exponential growth** 指数增长,
- $0 < b < 1$ gives **exponential decay** 指数衰减.

■ 4 Adding versus multiplying

A linear function **adds** the same amount each step; an exponential function **multiplies** by the same factor. That is why an exponential always overtakes a linear one in the end.

Watch out: exponential growth means multiplying by the same factor each step, so it starts slow but soon shoots up far past any straight line – that is the power of "compounding".

Practice

Try these in order. The methods are all above.

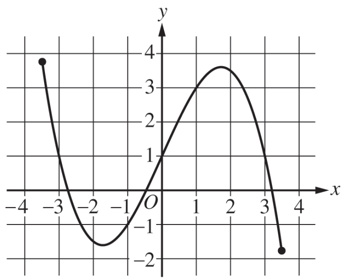
2.1 State the difference between an arithmetic and a geometric sequence. [2]

2.2 An arithmetic sequence has $a_0 = 4$ and $d = 3$. Find a_5. [2]

2.3 A geometric sequence has $g_0 = 5$ and $r = 2$. Find g_3. [2]

2.4 For $f(x) = 3 \cdot 2^x$, find $f(4)$. [2]

2.5 State whether $f(x) = 100 \cdot (0.5)^x$ shows growth or decay, and why. [2]



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

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Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$g(x) = e^{2x}$

$h(x) = \log_2(5x)$

i. Solve $g(x) = \frac{1}{e^6}$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the domain of h.

B. The functions j and k are given by

$j(x) = 7^{(3x+1)} \cdot 7^x$

$k(x) = \sin(2x)\sec x$

i. Rewrite $j(x)$ as an expression of the form $7^{(\text{expression})}$.

ii. Rewrite $k(x)$ as an expression in which $\sin x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = \tan^2(3x)$.

Find all input values for m in the interval $\left[0, \frac{\pi}{2}\right]$ that yield an output value of 1.

STOP

END OF EXAM

2. A person purchased a car at the end of the year 2019 ($t = 0$). The car's value decreases over time. At the end of the year 2020 ($t = 1$), the car was valued at 27.2 thousand dollars. At the end of the year 2025 ($t = 6$), the car was valued at 14.8 thousand dollars.

The value of the car can be modeled by the function V given by $V(t) = ab^t$, where $V(t)$ is the value of the car, in thousands of dollars, at time t, and t is the number of years since the end of 2019.

A.

i. Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $V(t)$.

ii. Find the values for a and b as decimal approximations.

B.

i. Use the given data to find the average rate of change of the value of the car, in thousands of dollars per year, from $t = 1$ to $t = 6$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

ii. Use the average rate of change found in part B (i) to estimate the value of the car, in thousands of dollars, at $t = 3$ years. Show the work that leads to your answer.

iii. The average rate of change found in part B (i) can be used to determine the secant line for the graph of V on the interval $1 \le t \le 6$. Let $A(t)$ represent the estimate of the value of the car, in thousands of dollars, at time t years, using the secant line. For $A(3)$, found in part B (ii), it can be shown that $A(3) > V(3)$.

In general, $A(t) > V(t)$ for all t, where $1 < t < 6$. Use the secant line and the graph of V to explain why this is true.

C. When the value of the car reaches 2 thousand dollars, the car's owner plans to donate it to an auto mechanic school. As a result, the car will immediately lose all of its value. Explain how this information can be used to determine a domain limitation for the model V.

END OF PART A

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i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

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i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

2 Logarithms and inverse functions – Part 2

Name: ____________ Class: ________ Date: ________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
inverse function	反函数	
logarithm	对数	

Recall

In Part A you met exponential growth. Now the tool that “undoes” it:

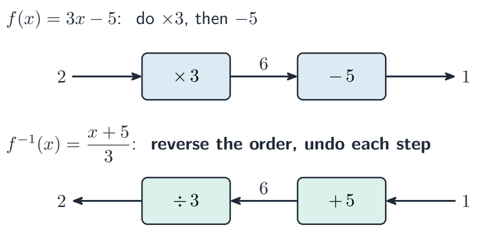
- You know that $2^3 = 8$.
- A **logarithm** answers the reverse question: what power gives 8?
- Working backwards through a function is called finding its inverse.

Each idea has a worked example before the Practice.

New ideas

1 Inverse functions

An **inverse function** 反函数 undoes what a function does: if $f(a) = b$, then $f^{-1}(b) = a$. Its graph is the reflection of f in the line $y = x$.



the inverse takes the output back to the input: $f(2) = 1$, so $f^{-1}(1) = 2$

2 Logarithms

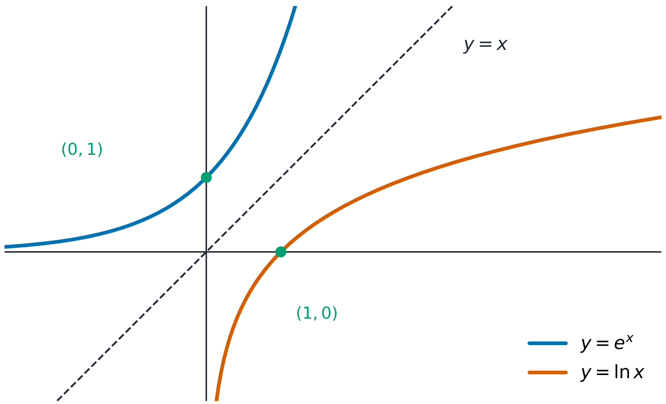
A **logarithm** 对数 answers "what exponent?":

$$\log_b c = a \quad \text{means exactly} \quad b^a = c.$$

Worked example. $\log_2 8 = 3$, because $2^3 = 8$. And $\log_{10} 1000 = 3$, because $10^3 = 1000$.

3 Logs undo exponentials

The logarithm is the **inverse of the exponential** – they cancel each other. So $\log_b(b^x) = x$ and $b^{\log_b x} = x$.



4 Solving exponential equations

Because they are inverses, you can use one to undo the other.

Worked example. Solve $2 \cdot 3^x = 54$: divide by 2 to get $3^x = 27 = 3^3$, so $x = 3$.

Watch out: you can only take the logarithm of a **positive** number – $\log_b(0)$ and logs of negatives are undefined. Always check that the thing inside a log stays positive.

Practice

Try these in order. The methods are all above.

2.1 State, in words, what $\log_b c = a$ means. [1]

2.2 Find $\log_2 16$. [2]

2.3 Find $\log_{10} 100$. [1]

2.4 Solve $5^x = 125$. [2]

2.5 Solve $3 \cdot 2^x = 48$. [3]

1. The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	-2	-1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

C.

i. Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}$(expression).

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

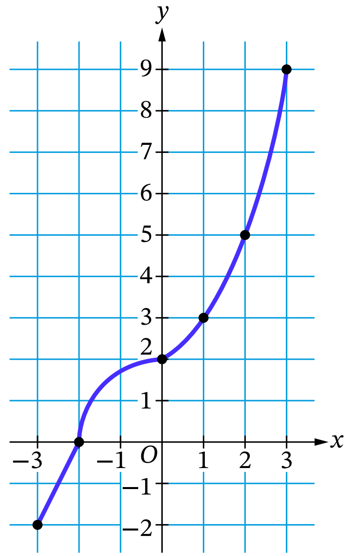
Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

1. The figure shows the graph of the increasing function f on its domain of $-3 \leq x \leq 3$.



Graph of f

The function g is given by $g(x) = -4.792 + \ln(6x - 6)$.

A.

i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

ii. Find all values of x for which $f(x) = 3$, or indicate that there are no such values.

B.

i. Find all values of x, as decimal approximations, for which $g(x) = -1.5$, or indicate that there are no such values.

ii. Determine the behavior of g as x-values decrease and get arbitrarily close to 1. Express your answer using the mathematical notation of a limit.

C.

i. Is the function f invertible?

ii. Give a reason for your answer in part C (i) based on properties of the function f. Refer to points on the graph of f in your reasoning.

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

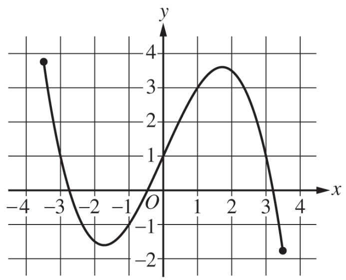
i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of m that yield an output value of 0.

STOP

END OF EXAM



Graph of f

1. The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined.

(ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(C) (i) Determine if f has an inverse function.

(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

2. On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t + 1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

END OF PART A

PRECALCULUS

SECTION II, Part B

Time—30 minutes

2 Questions

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

AP PRECALCULUS • HOLIDAY HOMEWORK 假期作业

3 Trig ratios and the unit circle – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
periodic	周期性	
period	周期	
unit circle	单位圆	
radians	弧度	

Recall

You have used trigonometry with right triangles:

- **SOH CAH TOA**: sine, cosine, and tangent of an angle.
- You have seen the smooth wave shape of a sine graph.
- Angles can be measured in degrees.

This sheet lines up the trig ideas the course builds on. Each idea has a worked example.

New ideas

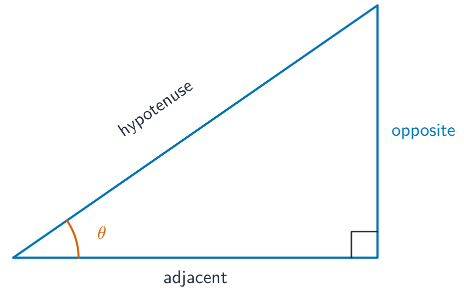
1 Periodic patterns

A pattern is **periodic** 周期性 if it repeats at regular steps. The **period** 周期 is how far you go before it repeats – like the tides, or day and night.

2 Right-triangle trig

For a right triangle, with an angle θ:

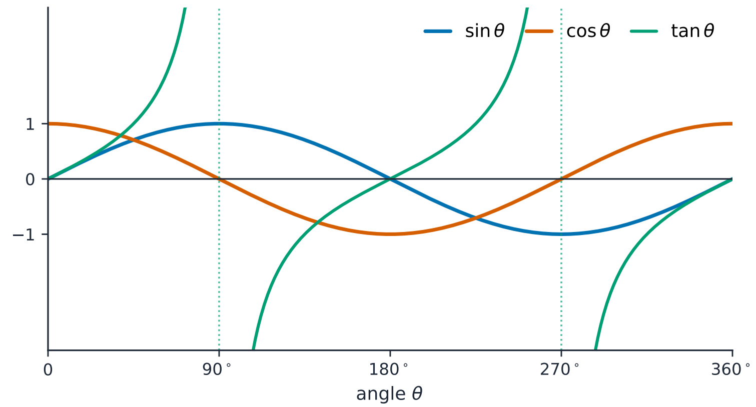
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}.$$



Worked example. In a right triangle with opposite = 3 and hypotenuse = 5: $\sin\theta = \frac{3}{5} = 0.6$.

3 The unit circle

On the **unit circle** 单位圆 (radius 1), the point at angle θ has coordinates $(\cos\theta, \sin\theta)$. Angles can be measured in **radians** 弧度 (a full turn is 2π).



4 Special angles

The values at $0°, 30°, 45°, 60°, 90°$ come up again and again – for example $\sin 30° = 0.5$ and $\cos 60° = 0.5$.

Watch out: $\sin\theta$ and $\cos\theta$ are always between -1 and 1 – they are ratios of a side to the hypotenuse (the longest side), so they can never exceed 1.

Practice

Try these in order. The methods are all above.

2.1 State what it means for a pattern to be periodic. [1]

2.2 Write the SOH CAH TOA ratios for sine, cosine, and tangent. [3]

2.3 A right triangle has opposite $= 4$ and hypotenuse $= 5$. Find $\sin\theta$. [2]

2.4 The same triangle has adjacent $= 3$. Find $\cos\theta$ and $\tan\theta$. [2]

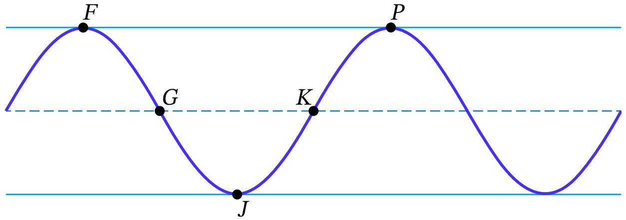
2.5 State the largest and smallest possible values of $\sin\theta$. [1]

3. For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

A. The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $\left(t, h(t)\right)$ for the five points: F, G, J, K, and P.



B. The function h can be written in the form $h(t) = a\sin\left(b(t+c)\right) + d$. Find values of constants a, b, c, and d.

C. Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

i. On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

ii. On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

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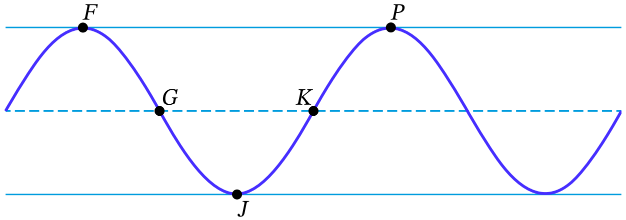

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Name: AP Precalculus: **2025**

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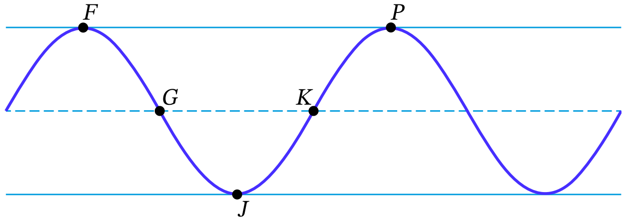

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3 Sine waves and inverse trig – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
amplitude	振幅			
midline	中线			
sinusoidal function	正弦型函数			
inverse trigonometric functions	反三角函数			

Recall

In Part A you met the sine and cosine ratios. Now their graphs and how to reshape them:

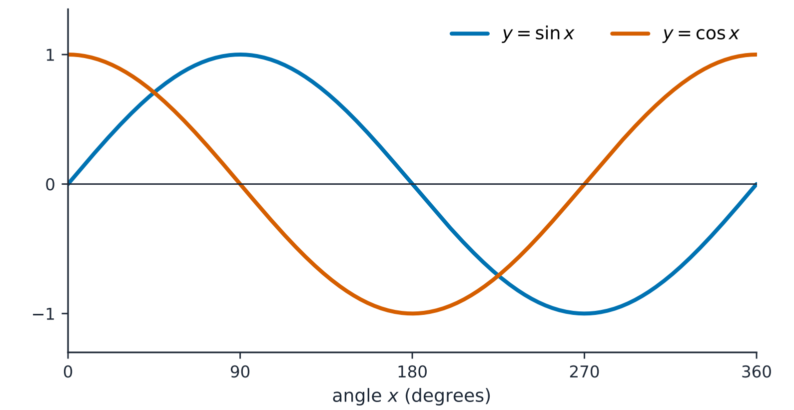
- The sine graph is a smooth, repeating wave.
- Real things like tides and daylight follow wave patterns.

Each idea has a worked example before the Practice.

New ideas

■ 1 Sine and cosine graphs

The graphs $y = \sin\theta$ and $y = \cos\theta$ are smooth waves that swing between -1 and 1 and repeat every 2π (one full turn).



■ 2 Amplitude, period, and midline

For a wave, three numbers describe its shape:

- the **amplitude** 振幅 = half the distance between the top and bottom,
- the **period** = how far along before it repeats,
- the **midline** 中线 = the horizontal centre line.

■ 3 Transforming a sine wave

A general **sinusoidal function** 正弦型函数 is

$$f(\theta) = a\sin(b\theta) + d,$$

where $|a|$ is the amplitude, the period is $\dfrac{2\pi}{|b|}$, and d is the midline.

Worked example. For $f(\theta) = 3\sin(2\theta) + 1$: amplitude = 3, period = $\dfrac{2\pi}{2} = \pi$, midline $y = 1$. So the maximum is $1 + 3 = 4$ and the minimum is $1 - 3 = -2$.

■ 4 Inverse trig

The **inverse trigonometric functions** 反三角函数 (arcsine, arccosine, arctangent) go the other way: give them a ratio and they return the angle.

Watch out: in $a\sin(b\theta) + d$, the number a changes the **amplitude** (height) while b changes the **period** (width). Don't confuse the two – b squeezes the wave sideways, not up and down.

Practice

Try these in order. The methods are all above.

2.1 State the range (lowest to highest value) of $y = \sin\theta$, and its period. [2]

2.2 For $f(\theta) = 5\sin\theta$, state the amplitude. [1]

2.3 For $f(\theta) = 2\sin(3\theta)$, find the amplitude and the period. [2]

2.4 For $f(\theta) = 4\sin(\theta) + 2$, state the midline, the maximum, and the minimum. [3]

2.5 State what an inverse trig function (like arcsine) does. [1]

4. **Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$
$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}$(expression).

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM

4. Directions:

- Unless otherwise specified, the domain of a function *f* is assumed to be the set of all real numbers *x* for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
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- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions *g* and *h* are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of *x* in the domain of *g*.

ii. Solve $h(x) = 3$ for values of *x* in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions *j* and *k* are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x(\csc^2 x - 1)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function *m* is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of *m* that yield an output value of 0.

STOP

END OF EXAM

4. Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x+3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

A. The functions g and h are given by

$$g(x) = 2\log_3 x$$
$$h(x) = 4\cos^2 x$$

i. Solve $g(x) = 4$ for values of x in the domain of g.

ii. Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.

B. The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$
$$k(x) = \frac{6}{\tan x\left(\csc^2 x - 1\right)}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

C. The function m is given by $m(x) = e^{2x} - e^x - 12$. Find all input values in the domain of m that yield an output value of 0.

STOP

END OF EXAM

4 Vectors – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
vector	向量			
magnitude	大小			

Recall

You may have met vectors already:

- A vector has a size **and** a direction, like a push of "5 newtons east".
- You have added movements: 3 steps east then 4 steps north.
- Pythagoras finds the straight-line distance of a right-angle journey.

This sheet lines up vectors. Each idea has a worked example.

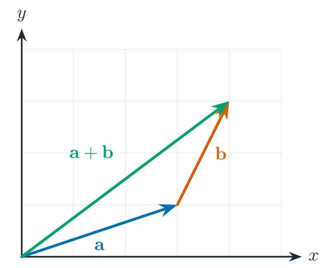
New ideas

1 What a vector is

A **vector** 向量 has both **magnitude** 大小 (length) and **direction**. We write it by its components, $\langle a, b\rangle$ – a across and b up.

2 Adding vectors

Add two vectors component by component: $\langle a, b\rangle + \langle c, d\rangle = \langle a + c,\ b + d\rangle$.



Worked example. $\langle 3, 1\rangle + \langle 2, 4\rangle = \langle 5, 5\rangle$.

3 Magnitude

The **magnitude** (length) of $\langle a, b\rangle$ is found with Pythagoras:

$$|\langle a, b\rangle| = \sqrt{a^2 + b^2}.$$

Worked example. $|\langle 3, 4\rangle| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

4 Scaling

Multiplying a vector by a number scales each component: $2\langle 3, 1\rangle = \langle 6, 2\rangle$ (same direction, twice as long).

Watch out: add vectors **component by component** – do not add their magnitudes. $\langle 3, 0\rangle + \langle 0, 4\rangle$ has magnitude 5 (by Pythagoras), not $3 + 4 = 7$.

Practice

Try these in order. The methods are all above.

2.1 State the two things a vector has. [2]

2.2 Find $\langle 2, 3\rangle + \langle 5, 1\rangle$. [2]

2.3 Find the magnitude of $\langle 6, 8\rangle$. [2]

2.4 Find $3\langle 2, 4\rangle$. [2]

2.5 A vector goes $\langle 5, 0\rangle$ and another $\langle 0, 12\rangle$. Find the magnitude of their sum. [3]

4 Matrices – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
matrix	矩阵			
determinant	行列式			
linear transformation	线性变换			

Recall

You may have seen numbers arranged in a grid (a table). A matrix is exactly that:

- A **matrix** is a rectangular grid of numbers.
- Matrices are used to store data and to move shapes around.

Each idea has a worked example before the Practice.

New ideas

1 What a matrix is

A **matrix** 矩阵 is a rectangular grid of numbers, for example $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ (2 rows, 2 columns).

2 The determinant

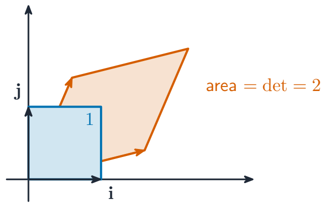
For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** 行列式 is

$$ad - bc.$$

Worked example. For $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, the determinant is $(3)(4) - (1)(2) = 12 - 2 = 10$.

■ **3 Matrices move the plane**

A 2×2 matrix acts as a **linear transformation** 线性变换 of the plane – it can stretch, rotate, or reflect points.



■ **4 The determinant as area**

The determinant tells you how much the transformation scales area. For example $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ stretches by 2 across and 3 up, so it multiplies every area by $2 \times 3 = 6$ – exactly its determinant.

Watch out: the determinant $ad - bc$ subtracts the off-diagonal product bc. A common slip is to compute $ad - bc$ as $ad + bc$ – remember the minus sign.

Practice

Try these in order. The methods are all above.

2.1 State what a matrix is. [1]

2.2 Find the determinant of $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. [2]

2.3 Find the determinant of $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$. [2]

2.4 State by what factor the matrix in 2.3 scales area. [1]

2.5 Find the determinant of $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$. [2]