

HOLIDAY HOMEWORK 假期作业

AP Precalculus

Exercise sheets, topic by topic.

每个单元：练习卷。

For class or self-study • no answer keys included.

Contents 目录

1	Functions and rates of change.....	3
2	Sequences and exponential functions.....	11
3	Trig ratios and the unit circle.....	19
4	Vectors.....	25

1 Functions and rates of change – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
function	函数	_____	_____	_____
domain	定义域	_____	_____	_____
range	值域	_____	_____	_____
increasing	递增	_____	_____	_____
decreasing	递减	_____	_____	_____
average rate of change	平均变化率	_____	_____	_____
secant line	割线	_____	_____	_____
linear	线性	_____	_____	_____
quadratic	二次	_____	_____	_____

Recall

You have already worked with functions and graphs:

- A function turns each input into one output, like $f(x) = x^2$.
- You have found the gradient (slope) of a straight line.
- You know a quadratic makes a U-shaped parabola.

This sheet lines up how functions change. Each idea has a worked example.

New ideas

■ 1 Functions, domain, and range

A **function** 函数 gives each input exactly one output. The allowed inputs are the **domain** 定义域; the outputs are the **range** 值域.

■ 2 Increasing and decreasing

A function is **increasing** 递增 where a bigger input gives a bigger output, and **decreasing** 递减 where a bigger input gives a smaller output.

■ 3 Average rate of change

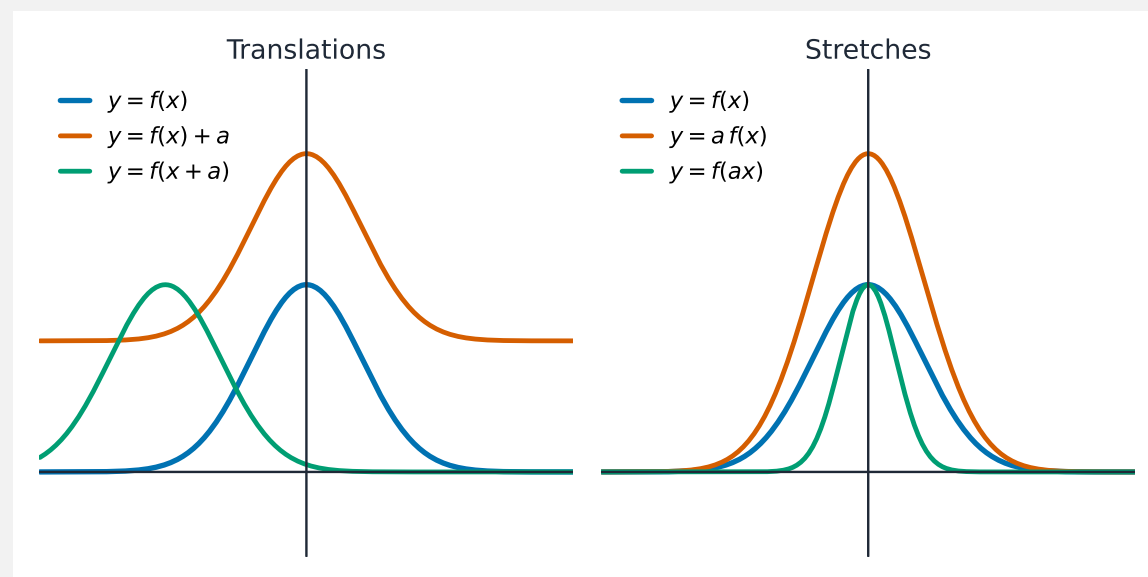
The **average rate of change** 平均变化率 over an interval $[a, b]$ is the change in output over the change in input:

$$\text{avg rate} = \frac{f(b) - f(a)}{b - a}.$$

This is just the gradient of the straight line (the **secant line** 割线) joining the two points.

Worked example. For $f(x) = x^2$ over $[1, 4]$: $\frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = 5$.

■ 4 Linear versus quadratic



For a **linear** 线性 function the average rate of change is **constant**; for a **quadratic** 二次 function it **changes** from interval to interval.

Watch out: the average rate of change is the slope of the line between two points – so for a curve it depends on **which** two points you pick. Only a straight line has the same rate everywhere.

Practice

Try these in order. The methods are all above.

2.1 State what the domain and the range of a function are. [2]

2.2 For $f(x) = 3x + 1$, find the average rate of change over $[0, 4]$. [2]

2.3 For $f(x) = x^2$, find the average rate of change over $[2, 5]$. [3]

2.4 State whether $f(x) = x^2$ is increasing or decreasing for $x > 0$. [1]

2.5 Explain why a linear function has the same average rate of change over every interval. [2]

1 Polynomials and rational functions

– Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
polynomial	多项式	_____	_____	_____
degree	次数	_____	_____	_____
zero	零点	_____	_____	_____
end behavior	末端行为	_____	_____	_____
rational function	有理函数	_____	_____	_____
vertical asymptote	垂直渐近线	_____	_____	_____
horizontal asymptote	水平渐近线	_____	_____	_____

Recall

In Part A you met functions and their rates of change. Now two important families:

- **Polynomials** like $x^3 - 2x + 1$ – sums of powers of x .
- **Rational functions** – one polynomial divided by another.

Each idea has a worked example before the Practice.

New ideas

■ 1 Polynomials

A **polynomial** 多项式 is a sum of powers of x , like $p(x) = 2x^3 - 5x + 1$. Its highest power is the **degree** 次数, and the term with that power is the **leading term**.

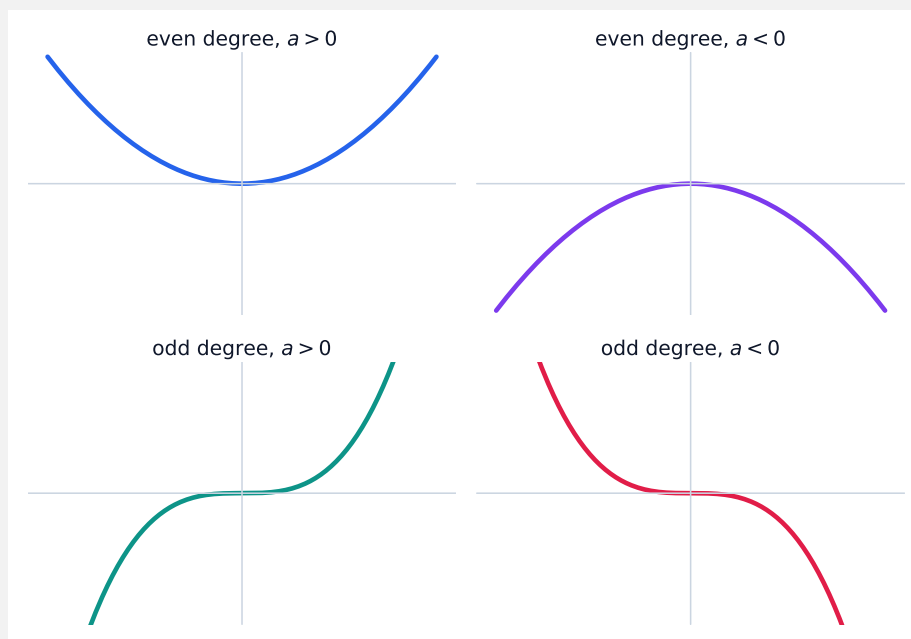
■ 2 Zeros

A **zero** 零点 (or root) of p is an input where $p(x) = 0$. Each real zero gives an **x -intercept** where the graph crosses the x -axis.

Worked example. $p(x) = (x - 2)(x + 3)$ has zeros at $x = 2$ and $x = -3$, so its graph crosses the x -axis at those two points.

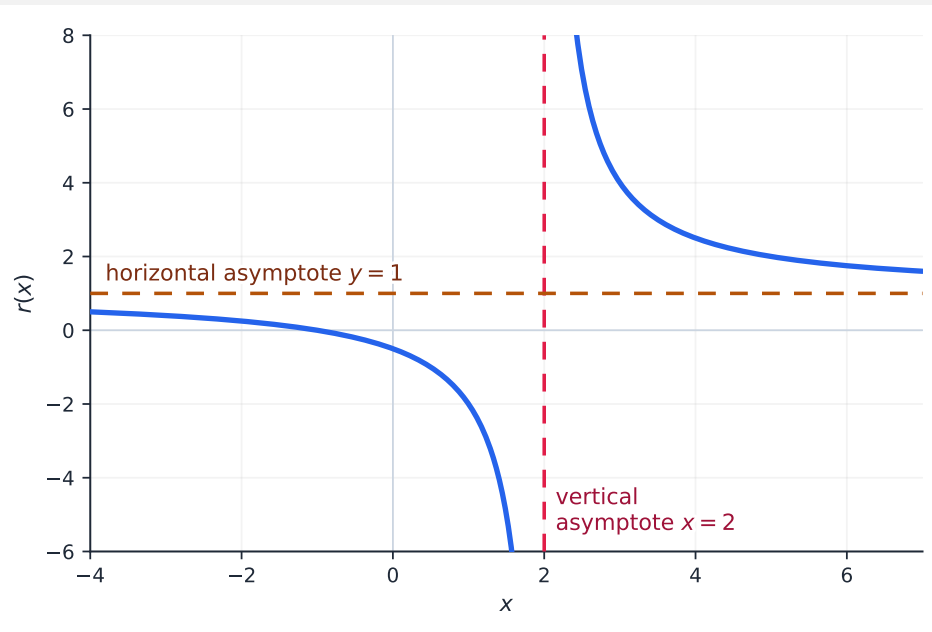
■ 3 End behavior

The **end behavior** 末端行为—where the graph heads as $x \rightarrow \pm\infty$ —is decided entirely by the leading term.



■ 4 Rational functions and asymptotes

A **rational function** 有理函数 is one polynomial over another. A **vertical asymptote** 垂直渐近线 appears where the denominator is zero (and the top is not), and a **horizontal asymptote** 水平渐近线 describes where the graph levels off far out.



Worked example. For $r(x) = \frac{1}{x-2}$, the denominator is zero at $x = 2$, so there is a vertical asymptote there; far out the value tends to 0, so $y = 0$ is a horizontal asymptote.

Watch out: a graph can **cross** a horizontal asymptote but never crosses a **vertical** one –the function shoots off to $\pm\infty$ as it approaches a vertical asymptote.

Practice

Try these in order. The methods are all above.

2.1 State the degree of $p(x) = 4x^3 - 2x^2 + 7$. [1]

2.2 Find the zeros of $p(x) = (x - 1)(x + 4)$. [2]

2.3 State where the graph of $p(x) = (x - 1)(x + 4)$ crosses the x -axis. [1]

2.4 For $r(x) = \frac{1}{x-3}$, state the vertical asymptote. [1]

2.5 For $r(x) = \frac{5}{x+1}$, state the horizontal asymptote as $x \rightarrow \pm\infty$. [1]

2 Sequences and exponential functions

– Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
sequence	数列	_____	_____	_____
arithmetic sequence	se- 等差数列	_____	_____	_____
common difference	公差	_____	_____	_____
geometric sequence	se- 等比数列	_____	_____	_____
common ratio	公比	_____	_____	_____
exponential function	指数函数	_____	_____	_____
exponential growth	指数增长	_____	_____	_____
exponential decay	指数衰减	_____	_____	_____

Recall

You have seen number patterns and powers before:

- Sequences like 2, 4, 6, 8, ... (adding) or 2, 4, 8, 16, ... (doubling).
- Powers like $2^3 = 8$.
- Some things grow faster and faster –like money earning interest.

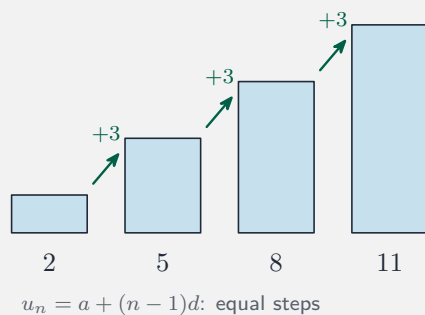
This sheet lines up sequences and exponential growth. Each idea has a worked example.

New ideas

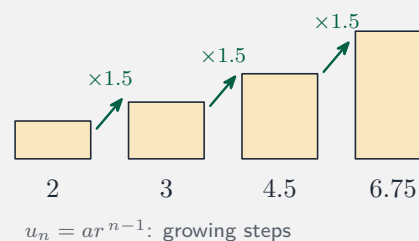
■ 1 Two kinds of sequence

A **sequence** 数列 is an ordered list of numbers. Two important types:

arithmetic: add d each step



geometric: multiply by r each step



- an **arithmetic sequence** 等差数列 **adds** a fixed **common difference** 公差 d each step,
- a **geometric sequence** 等比数列 **multiplies** by a fixed **common ratio** 公比 r each step.

Worked example. Arithmetic with $a_0 = 3$, $d = 5$: the terms are 3, 8, 13, 18, ...
Geometric with $g_0 = 2$, $r = 3$: 2, 6, 18, 54, ...

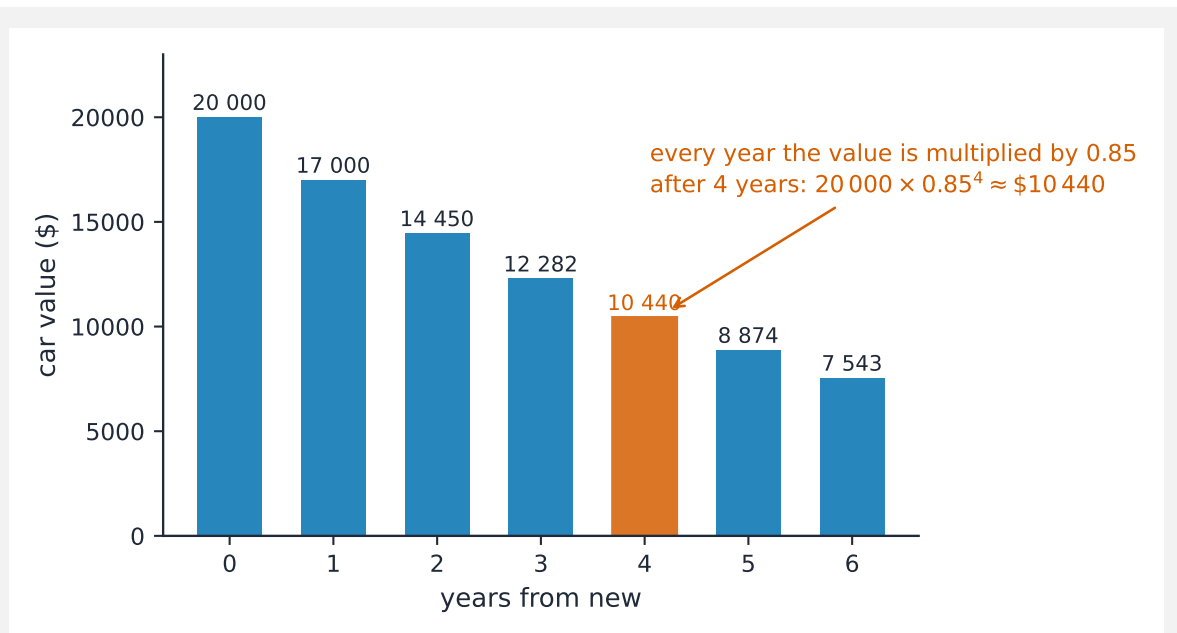
■ 2 Finding a term

For arithmetic, $a_n = a_0 + dn$; for geometric, $g_n = g_0 r^n$.

Worked example. $a_4 = 3 + 5(4) = 23$; $g_4 = 2 \cdot 3^4 = 162$.

■ 3 Exponential functions

An **exponential function** 指数函数 is $f(x) = ab^x$ —a starting value a multiplied by the **base** b repeatedly.



- $b > 1$ gives **exponential growth** 指数增长,
- $0 < b < 1$ gives **exponential decay** 指数衰减.

■ 4 Adding versus multiplying

A linear function **adds** the same amount each step; an exponential function **multiplies** by the same factor. That is why an exponential always overtakes a linear one in the end.

Watch out: exponential growth means multiplying by the same factor each step, so it starts slow but soon shoots up far past any straight line –that is the power of “compounding”.

Practice

Try these in order. The methods are all above.

2.1 State the difference between an arithmetic and a geometric sequence. [2]

2.2 An arithmetic sequence has $a_0 = 4$ and $d = 3$. Find a_5 . [2]

2.3 A geometric sequence has $g_0 = 5$ and $r = 2$. Find g_3 . [2]

2.4 For $f(x) = 3 \cdot 2^x$, find $f(4)$. [2]

2.5 State whether $f(x) = 100 \cdot (0.5)^x$ shows growth or decay, and why. [2]

2 Logarithms and inverse functions – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
inverse function	反函数	_____
logarithm	对数	_____

Recall

In Part A you met exponential growth. Now the tool that "undoes" it:

- You know that $2^3 = 8$.
- A **logarithm** answers the reverse question: what power gives 8?
- Working backwards through a function is called finding its inverse.

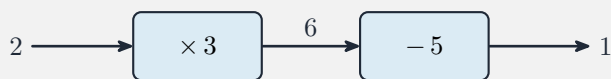
Each idea has a worked example before the Practice.

New ideas

■ 1 Inverse functions

An **inverse function** 反函数 undoes what a function does: if $f(a) = b$, then $f^{-1}(b) = a$. Its graph is the reflection of f in the line $y = x$.

$f(x) = 3x - 5$: do $\times 3$, then -5



$f^{-1}(x) = \frac{x+5}{3}$: **reverse the order, undo each step**



the inverse takes the output back to the input: $f(2) = 1$, so $f^{-1}(1) = 2$

■ 2 Logarithms

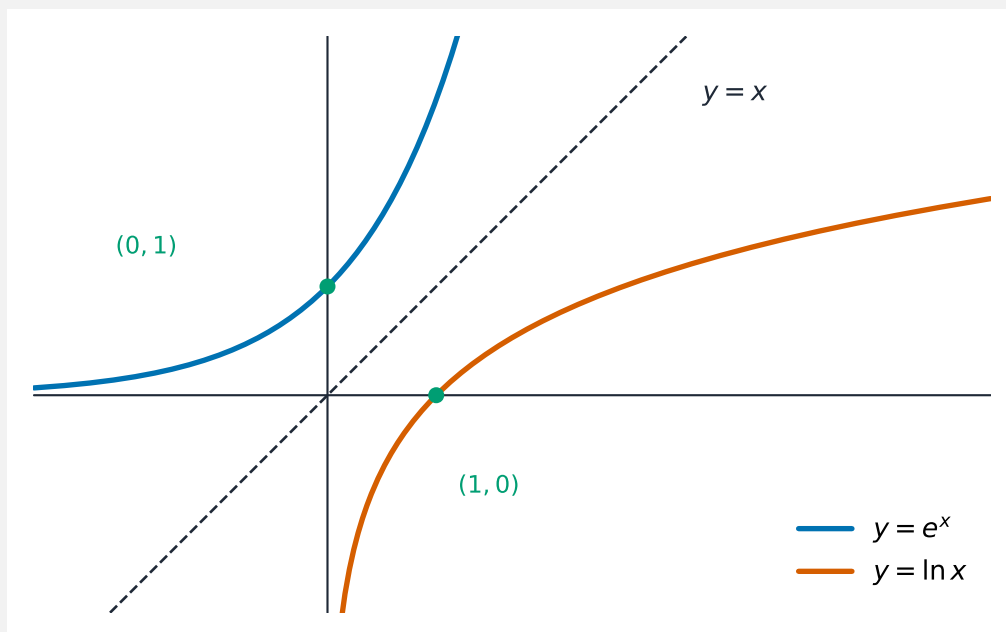
A **logarithm** 对数 answers "what exponent?":

$$\log_b c = a \text{ means exactly } b^a = c.$$

Worked example. $\log_2 8 = 3$, because $2^3 = 8$. And $\log_{10} 1000 = 3$, because $10^3 = 1000$.

■ 3 Logs undo exponentials

The logarithm is the **inverse of the exponential**—they cancel each other. So $\log_b(b^x) = x$ and $b^{\log_b x} = x$.



■ 4 Solving exponential equations

Because they are inverses, you can use one to undo the other.

Worked example. Solve $2 \cdot 3^x = 54$: divide by 2 to get $3^x = 27 = 3^3$, so $x = 3$.

Watch out: you can only take the logarithm of a **positive** number— $\log_b(0)$ and logs of negatives are undefined. Always check that the thing inside a log stays positive.

Practice

Try these in order. The methods are all above.

2.1 State, in words, what $\log_b c = a$ means. [1]

2.2 Find $\log_2 16$. [2]

2.3 Find $\log_{10} 100$. [1]

2.4 Solve $5^x = 125$. [2]

2.5 Solve $3 \cdot 2^x = 48$. [3]

3 Trig ratios and the unit circle – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
periodic	周期性	_____
period	周期	_____
unit circle	单位圆	_____
radians	弧度	_____

Recall

You have used trigonometry with right triangles:

- **SOH CAH TOA**: sine, cosine, and tangent of an angle.
- You have seen the smooth wave shape of a sine graph.
- Angles can be measured in degrees.

This sheet lines up the trig ideas the course builds on. Each idea has a worked example.

New ideas

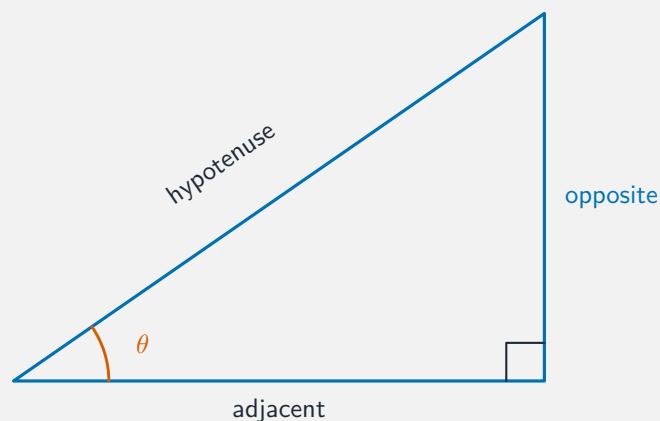
■ 1 Periodic patterns

A pattern is **periodic** 周期性 if it repeats at regular steps. The **period** 周期 is how far you go before it repeats –like the tides, or day and night.

■ 2 Right-triangle trig

For a right triangle, with an angle θ :

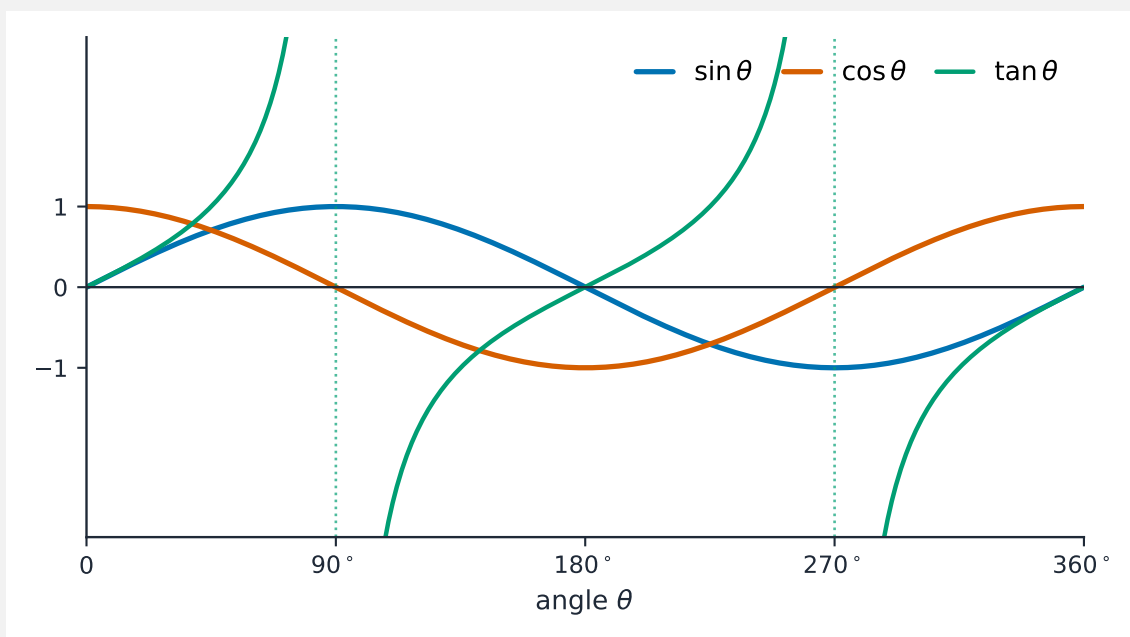
$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}.$$



Worked example. In a right triangle with opposite = 3 and hypotenuse = 5: $\sin \theta = \frac{3}{5} = 0.6$.

■ 3 The unit circle

On the **unit circle** 单位圆 (radius 1), the point at angle θ has coordinates $(\cos \theta, \sin \theta)$. Angles can be measured in **radians** 弧度 (a full turn is 2π).



■ 4 Special angles

The values at $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ come up again and again—for example $\sin 30^\circ = 0.5$ and $\cos 60^\circ = 0.5$.

Watch out: $\sin \theta$ and $\cos \theta$ are always between -1 and 1 —they are ratios of a side to the hypotenuse (the longest side), so they can never exceed 1.

Practice

Try these in order. The methods are all above.

2.1 State what it means for a pattern to be periodic. [1]

2.2 Write the SOH CAH TOA ratios for sine, cosine, and tangent. [3]

2.3 A right triangle has opposite = 4 and hypotenuse = 5. Find $\sin \theta$. [2]

2.4 The same triangle has adjacent = 3. Find $\cos \theta$ and $\tan \theta$. [2]

2.5 State the largest and smallest possible values of $\sin \theta$. [1]

3 Sine waves and inverse trig – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
amplitude	振幅	_____
midline	中线	_____
sinusoidal function	正弦型函数	_____
inverse trigonometric functions	反三角函数	_____

Recall

In Part A you met the sine and cosine ratios. Now their graphs and how to reshape them:

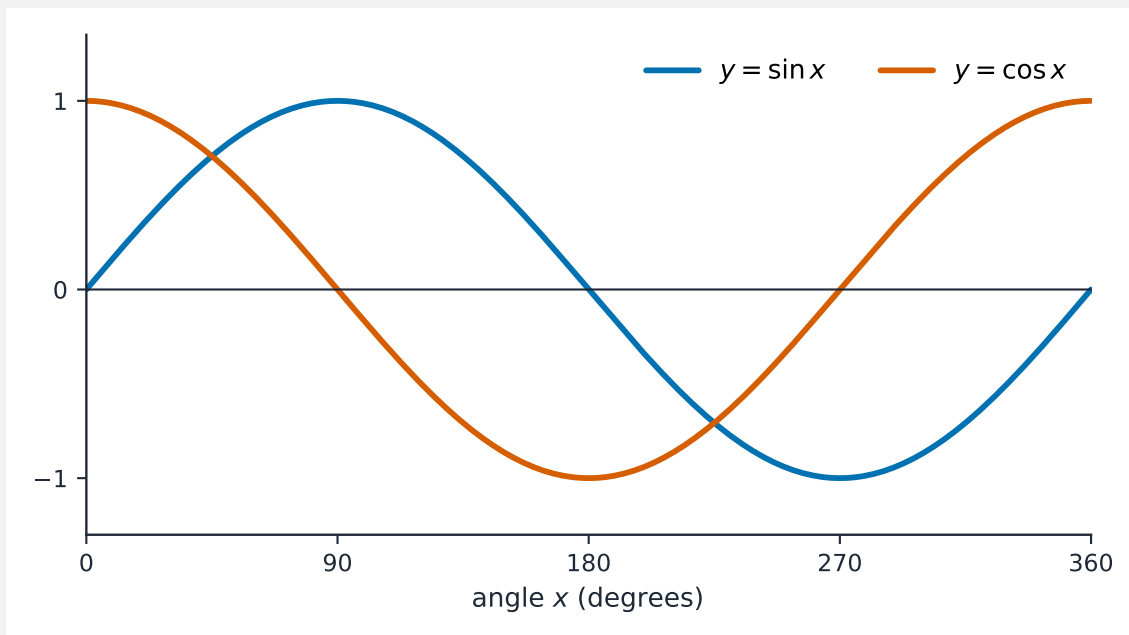
- The sine graph is a smooth, repeating wave.
- Real things like tides and daylight follow wave patterns.

Each idea has a worked example before the Practice.

New ideas

■ 1 Sine and cosine graphs

The graphs $y = \sin \theta$ and $y = \cos \theta$ are smooth waves that swing between -1 and 1 and repeat every 2π (one full turn).



■ 2 Amplitude, period, and midline

For a wave, three numbers describe its shape:

- the **amplitude** 振幅 = half the distance between the top and bottom,
- the **period** = how far along before it repeats,
- the **midline** 中线 = the horizontal centre line.

■ 3 Transforming a sine wave

A general **sinusoidal function** 正弦型函数 is

$$f(\theta) = a \sin(b\theta) + d,$$

where $|a|$ is the amplitude, the period is $\frac{2\pi}{|b|}$, and d is the midline.

Worked example. For $f(\theta) = 3 \sin(2\theta) + 1$: amplitude = 3, period = $\frac{2\pi}{2} = \pi$, midline $y = 1$. So the maximum is $1 + 3 = 4$ and the minimum is $1 - 3 = -2$.

■ 4 Inverse trig

The **inverse trigonometric functions** 反三角函数 (arcsine, arccosine, arctangent) go the other way: give them a ratio and they return the angle.

Watch out: in $a \sin(b\theta) + d$, the number a changes the **amplitude** (height) while b changes the **period** (width). Don't confuse the two $-b$ squeezes the wave sideways, not up and down.

Practice

Try these in order. The methods are all above.

2.1 State the range (lowest to highest value) of $y = \sin \theta$, and its period. [2]

2.2 For $f(\theta) = 5 \sin \theta$, state the amplitude. [1]

2.3 For $f(\theta) = 2 \sin(3\theta)$, find the amplitude and the period. [2]

2.4 For $f(\theta) = 4 \sin(\theta) + 2$, state the midline, the maximum, and the minimum. [3]

2.5 State what an inverse trig function (like arcsine) does. [1]

4 Vectors – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word *three times*. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
vector	向量	_____
magnitude	大小	_____

Recall

You may have met vectors already:

- A vector has a size **and** a direction, like a push of "5 newtons east".
- You have added movements: 3 steps east then 4 steps north.
- Pythagoras finds the straight-line distance of a right-angle journey.

This sheet lines up vectors. Each idea has a worked example.

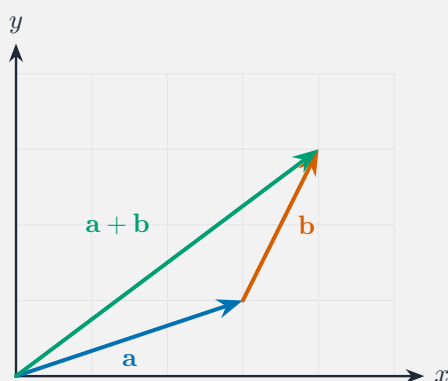
New ideas

■ 1 What a vector is

A **vector** 向量 has both **magnitude** 大小 (length) and **direction**. We write it by its components, $\langle a, b \rangle$ – a across and b up.

■ 2 Adding vectors

Add two vectors component by component: $\langle a, b \rangle + \langle c, d \rangle = \langle a + c, b + d \rangle$.



Worked example. $\langle 3, 1 \rangle + \langle 2, 4 \rangle = \langle 5, 5 \rangle$.

■ 3 Magnitude

The **magnitude** (length) of $\langle a, b \rangle$ is found with Pythagoras:

$$|\langle a, b \rangle| = \sqrt{a^2 + b^2}.$$

Worked example. $|\langle 3, 4 \rangle| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

■ 4 Scaling

Multiplying a vector by a number scales each component: $2\langle 3, 1 \rangle = \langle 6, 2 \rangle$ (same direction, twice as long).

Watch out: add vectors **component by component** –do not add their magnitudes. $\langle 3, 0 \rangle + \langle 0, 4 \rangle$ has magnitude 5 (by Pythagoras), not $3 + 4 = 7$.

Practice

Try these in order. The methods are all above.

2.1 State the two things a vector has. [2]

2.2 Find $\langle 2, 3 \rangle + \langle 5, 1 \rangle$. [2]

2.3 Find the magnitude of $\langle 6, 8 \rangle$.

[2]

2.4 Find $3\langle 2, 4 \rangle$.

[2]

2.5 A vector goes $\langle 5, 0 \rangle$ and another $\langle 0, 12 \rangle$. Find the magnitude of their sum.

[3]

4 Matrices – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
matrix	矩阵	_____
determinant	行列式	_____
linear transformation	线性变换	_____

Recall

You may have seen numbers arranged in a grid (a table). A matrix is exactly that:

- A **matrix** is a rectangular grid of numbers.
- Matrices are used to store data and to move shapes around.

Each idea has a worked example before the Practice.

New ideas

■ 1 What a matrix is

A **matrix** 矩阵 is a rectangular grid of numbers, for example $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$ (2 rows, 2 columns).

■ 2 The determinant

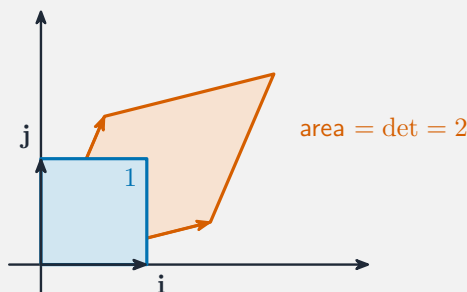
For a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** 行列式 is

$$ad - bc.$$

Worked example. For $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, the determinant is $(3)(4) - (1)(2) = 12 - 2 = 10$.

■ 3 Matrices move the plane

A 2×2 matrix acts as a **linear transformation** 线性变换 of the plane –it can stretch, rotate, or reflect points.



■ 4 The determinant as area

The determinant tells you how much the transformation scales area. For example $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ stretches by 2 across and 3 up, so it multiplies every area by $2 \times 3 = 6$ – exactly its determinant.

Watch out: the determinant $ad - bc$ subtracts the off-diagonal product bc . A common slip is to compute $ad - bc$ as $ad + bc$ –remember the minus sign.

Practice

Try these in order. The methods are all above.

2.1 State what a matrix is.

[1]

2.2 Find the determinant of $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$.

[2]

2.3 Find the determinant of $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$. [2]

2.4 State by what factor the matrix in 2.3 scales area. [1]

2.5 Find the determinant of $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$. [2]

