Question 4: Qualitative Quantitative Translation (QQT) — 8 points

A

For indicating $f_D < f_R$ — **Point A1**

For a justification that compares **one** of the following: — **Point A2**

- The motions of the disk and the ring using translational kinematics
- The motions of the disk and the ring using rotational kinematics
- The rotational inertias of the disk and the ring

For a justification that includes **one** of the following: — **Point A3**

- Reasoning that attempts Newton's second law in translational form
- Reasoning that attempts Newton's second law in rotational form
- Reasoning that attempts conservation of energy

Example Response

The ring has a greater rotational inertia because it has more mass distributed towards the edge. So the ring travels farther and has less acceleration down the ramp. Because the ring and the disk have the same mass, the gravitational forces exerted on the shapes are the same. From Newton's second law, the ring must have less net force down the ramp and more friction up the ramp.

B

For a multistep derivation that includes Newton's second law in both translational and rotational forms — **Point B1**

For indicating opposite signs for the gravitational force and frictional force in an expression for Newton's second law — **Point B2**

For using the relationship $a = r\alpha$ in an attempt to solve a system of equations — **Point B3**

Scoring Note: Responses that include conservation of energy may earn full credit.

Example Response

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}}$$

$$\left(\frac{a}{R}\right) = \frac{fR}{I}$$

$$a = \frac{fR^2}{I}$$

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}}$$

$$f - Mg\sin\theta = -Ma$$

$$f - Mg\sin\theta = -M\frac{fR^2}{I}$$

$$f + M\frac{fR^2}{I} = Mg\sin\theta$$

$$f\left(1 + \frac{MR^2}{I}\right) = Mg\sin\theta$$

$$f = \frac{Mg\sin\theta}{1 + \frac{MR^2}{I}}$$

C	For indicating "Equal to"	**Point C1**
	For a justification that includes that kinetic friction is only dependent on the surfaces (or equivalently, the coefficient of kinetic friction between those surfaces) and normal force	**Point C2**
	Example Response *The forces of kinetic friction are equal. If slipping, the friction is kinetic, and the force of kinetic friction on the hoop and the disk are the same because the coefficient of kinetic friction is only dependent on the surfaces, and the normal forces on both are the same.*	

Question 3: Free-Response Question — 15 points

(a) For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting γx^2 into the above equation — **1 point**

$$I = \int x^2 \left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma \left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

Total for part (a) — 2 points

(b) For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting γx^2 into the numerator of the above equation — **1 point**

For correctly substituting M into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

Total for part (b) — 3 points

(c)	For selecting "Greater than" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (c)

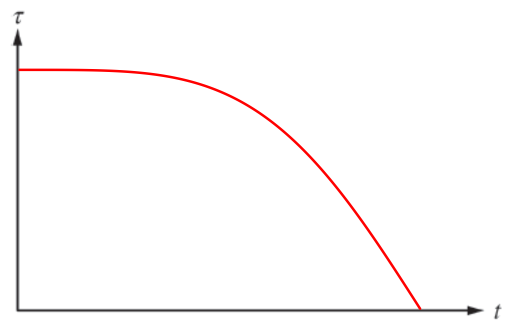
Because more of the mass of the rod is at the end of the rod opposite point P, more mass is concentrated away from the axis of rotation; thus, the rotational inertia of the rod would be greater around point P than around its center of mass.

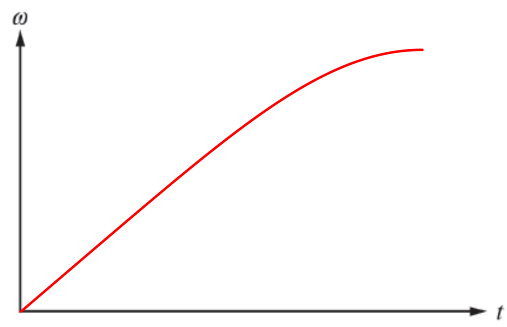
OR

According to the parallel axis theorem, $I = I_{cm} + md^2$, *if the axis is at a position away from the center of mass, the rotational inertia is larger than if the axis were at the center of mass.*

Total for part (c) 2 points

(d)	For a concave down curve that decreases to zero for the graph of τ as a function of t	**1 point**
	For a concave down curve that approaches horizontal for the graph of ω as a function of t	**1 point**
	For consistency between the two graphs	**1 point**





Total for part (d) 3 points

(e)	For selecting "Decreases" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (e)

As the rod rotates downward, the angle θ *in the torque equation* $\tau = rF\sin\theta$ *decreases. Thus, the torque on the rod decreases.*

OR

As the rod rotates downward, the lever arm between point P and the rod's center of mass continues to decrease; thus, the torque on the rod decreases.

Total for part (e) 2 points

(f)	For using conservation of energy to calculate the speed of the rotating rod $U_i + K_i = U_f + K_f$ $U_i + 0 = 0 + K_f$ $U_i = K_f$	**1 point**
	For correctly substituting into the above equation $mgh_i = \frac{1}{2}I\omega_f^2$	**1 point**
	For correctly solving for the linear speed of point S $Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$ $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m})} = 4.9\ \text{m/s}$	**1 point**
	Total for part (f)	**3 points**
	Total for question 3	**15 points**

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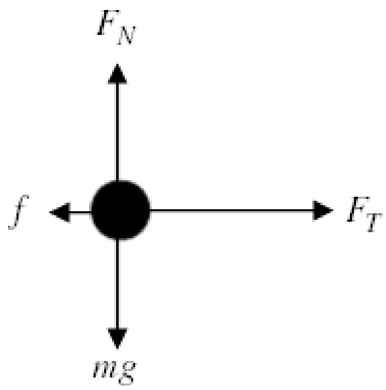
Question

15 points total | **Distribution of points**

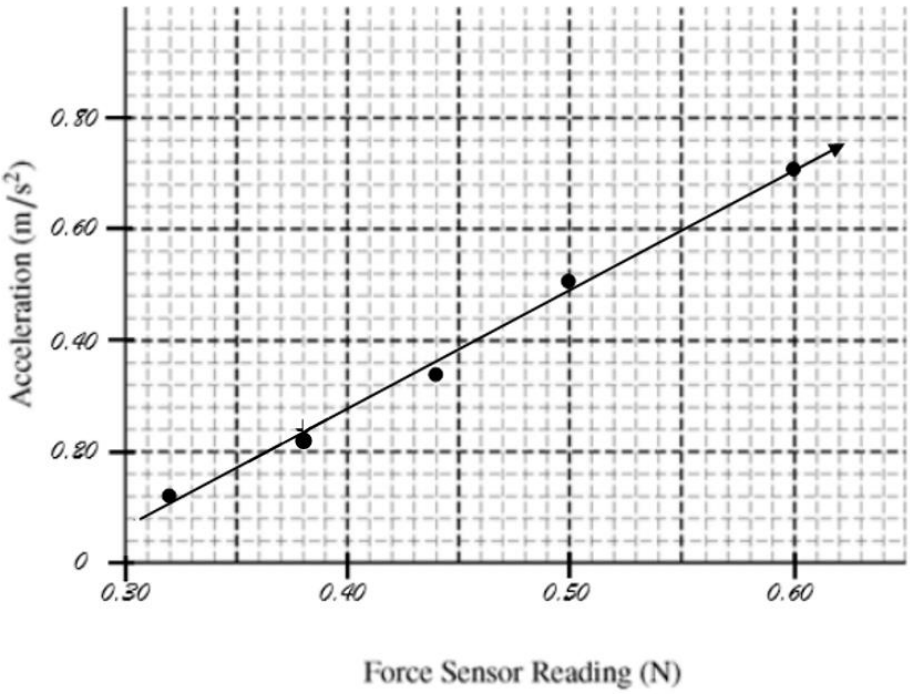
3 points



For correctly drawing and labeling the force of tension	1 point
For correctly drawing and labeling the force of friction	1 point
For correctly drawing and labeling both forces in the vertical direction	1 point

Note: A maximum of two points may be earned if there are any extraneous vectors.

3 points



For a correct scale that uses more than half the grid	1 point
For correctly plotting the given data	1 point
For drawing a straight line consistent with the given data	1 point

Note: Full credit can be earned if the axes are switched.

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

2 points

For correctly calculating slope using the best-fit straight line and not data points — 1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.70 - 0.16)}{(0.60 - 0.35)}\text{kg}^{-1} = 2.16\ \text{kg}^{-1}$$

Note: Linear regression gives slope $= 2.12\ \text{kg}^{-1}$.

For correctly calculating the mass of the cart using the slope — 1 point

$$m = \frac{1}{\text{slope}} = \frac{1}{\left(2.16\ \text{kg}^{-1}\right)}$$

Correct answer:

$m = 0.463\ \text{kg}$ (Note: linear regression gives $m = 0.472\ \text{kg}$)

1 point

For an answer with correct units consistent with the x-intercept of the graph from (b) i. — 1 point

$f = 0.272\ \text{N}$

(

1 point

plying Newton's second law and substituting the values from part (b)

$$\sum F = ma = F_a - f$$

$$a = \frac{F_a - f}{m} = \frac{0.45\ \text{N} - 0.272\ \text{N}}{0.463\ \text{kg}}$$

For an answer with correct units consistent with part (b), either from the graph or calculated using the mass and frictional force. — 1 point

$a = 0.376\ \text{m/s}^2$

SCORING GUIDELINES

Question 1 (continued)

Distribution of points

3 points

For using a correct equation to solve for the speed of the cart when the string breaks, with an acceleration consistent with part (c) i. — 1 point

$v_2 = v_1 + at$

$v_2 = 0 + \left(0.376\ \text{m/s}^2\right)(2.0\ \text{s})$

$v_2 = 0.752\ \text{m/s}$

For recognizing that the acceleration of the cart after the string breaks is due to the frictional force determined in part (b) — 1 point

Use a correct equation to solve for the time for the cart to stop after the string breaks

$v_2 = v_1 + at$

$0 = v_1 - \dfrac{f}{m}t$

For using the final velocity before the string breaks as the initial velocity for the cart stopping in the correct equation for time — 1 point

$t = \dfrac{mv_1}{f}$

$t = \dfrac{(0.463\ \text{kg})(0.752\ \text{m/s})}{(0.272\ \text{N})}$

Correct answer

$t = 1.28\ \text{s}$

Alternate solution — *Alternate points*

For setting the magnitude of the impulse before the string breaks equal to the magnitude of the impulse after the string breaks — *1 point*

$F_1 t_1 = F_2 t_2$

For correctly using the proper force (e.g., the tension force minus the friction force) for F_1 — *1 point*

For correctly using the proper force (e.g., the friction force) for F_2 — *1 point*

Correct answer

$t = 1.28\ \text{s}$

1 point

For selecting “Equal to” — 1 point

1 point

For selecting “Greater than” — 1 point

SCORING GUIDELINES

Question

15 points total **Distribution of points**

2 points

For correctly drawing and labeling vectors on the block of mass $2M$ — 1 point

For correctly drawing and labeling vectors on the block of mass $3M$ using symbols for the vectors that are physically correct and different from those on the block of mass $2M$ — 1 point

Note: A maximum of one point can be earned if there are any extraneous vectors.

2 points

Using a proper expression for conservation of momentum

$p_1 = p_2$

For correctly substituting into the above equation — 1 point

$3Mv_0 = (3M + 2M)v_f$

For a correct answer — 1 point

$v_f = \frac{3}{5}v_0$

(1 point

Using a proper expression for kinetic energy of the two-block system

$K = \frac{1}{2}mv^2$

$K = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2$

For an answer consistent with part (b) — 1 point

$K = \frac{9}{10}Mv_0^2$

SCORING GUIDELINES

Question continued)

Distribution of points

4 points

For a correct expression of the conservation of energy — 1 point

$\Delta K_{system} + \Delta U_{system} = 0$

$K_0 = U_{final}$

For attempting to integrate the spring force equation — 1 point

$\frac{9}{10}Mv_0^2 = -\int_{x_1}^{x_2} -Bx^3 dx$

$\frac{9}{10}Mv_0^2 = \int_{x_1}^{x_2} Bx^3 dx$

For using the correct limits of integration or an appropriate constant of integration — 1 point

$\frac{9}{10}Mv_0^2 = \int_0^D Bx^3 dx$

$\frac{9}{10}Mv_0^2 = \left[\frac{Bx^4}{4}\right]_0^D$

For an answer consistent with the speed from (b) or the kinetic energy from part (c) — 1 point

$D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$

2 points

For selecting the correct answer "Right" with a reasonable attempt at a justification — 1 point

If the incorrect selection is made, no points are earned for the justification.

For an indication that at maximum compression the block of mass $2M$ has an acceleration to the right due to the forces acting on the block of mass $2M$ or an acceleration to the right due to the external spring force acting on the system of blocks — 1 point

Example: At maximum compression the two-block system is instantaneously at rest. The only horizontal external force acting on the system is due to the spring. This force is directed to the right. The system and therefore the block of mass $2M$ is accelerated to the right, which implies that the net force acting on the block of mass $2M$ is also to the right.

SCORING GUIDELINES

Question continued)

Distribution of points

2 points

e magnitude of the net force is greater on the block of mass $3M$.

If the incorrect selection is made, no points are earned for the justification.

For an indication that both blocks will have the same acceleration — 1 point

For a correct justification for why the net force is greater on the block of mass $3M$ — 1 point

Example:

Because the blocks stick together, both blocks must have the same acceleration. Because the block of mass $3M$ has more mass, the net force on it must be greater than the net force on the block of mass $2M$.

2 points

For selecting the correct answer "No," with a reasonable attempt at a justification — 1 point

If the incorrect selection is made, no points are earned for the justification.

If the correct answer is selected without any justification, the point is not earned for the selection.

For an indication that the spring does not apply a linear force and that simple harmonic motion is the resulting motion of a linear restoring force — 1 point

$\left(a = -\frac{k}{m}\Delta x\right)$

Example:

Because the blocks are sticking together and are attached to the spring, the spring will apply a restoring force to the blocks. However, because the restoring force exerted by a nonlinear spring is not proportional to the blocks' displacement from equilibrium, the blocks do not exhibit simple harmonic motion.

2016 SCORING GUIDELINES

Question

15 points total | **Distribution of points**

3 points

For an indication that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

correct substitution for x in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for k — 1 point

$k\left(\frac{d}{2}\right) = m(2d)\omega^2$

rect answer:

$k = 4m\omega^2$

1 point

Substitute values for the block into the equation for rotational inertia

$I = \sum mr^2 = m(2d)^2$

For a correct answer or an answer consistent with the radius used in part (a) — 1 point

$I = 4md^2$

2 points

For an indication that the rotational inertia of the system must include the platform, the rod, and the object — 1 point

$I = I_P + I_R + I_O$

$I = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$

For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i. — 1 point

$I = \frac{5m(2d)^2}{2} + \frac{3md^2}{3} + 4md^2$

$I = (10 + 1 + 4)md^2$

rect answer:

$I = 15md^2$

1 point

Using a correct expression of angular momentum

$L = I\omega$

For an answer consistent with the answer from part (b) ii. — 1 point

$L = 15md^2\omega$

SCORING GUIDELINES

Question continued)

Distribution of points

3 points

For indicating that the spring force equals the mass times the centripetal acceleration — 1 point

$F_S = F_C = ma_C$

$kx = mr\omega^2$

For a correct substitution for k, or a substitution consistent with part (a), in the equation — 1 point

For a correct substitution for r in the equation that can be used to solve for x — 1 point

$\left(4m\omega^2\right)x = m(d + x)\omega^2$

$4x = d + x$

$3x = d$

Correct answer:

$x = d/3$

2 points

electing "Decreasing" — 1 point

the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: The rotational inertia I of the system decreases while the angular velocity w stays the same. Because the angular momentum is Iw, it decreases.

2 points

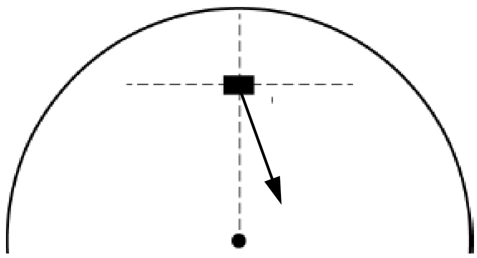
For selecting "Negative" — 1 point

If the wrong selection is made, no points are earned for the justification.

For a correct justification — 1 point

Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.

1 point



n arrow starting on the box and pointing down and to the right — 1 point