

Question 4

4. A uniform disk and ring, each of mass M and radius R , roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v .

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

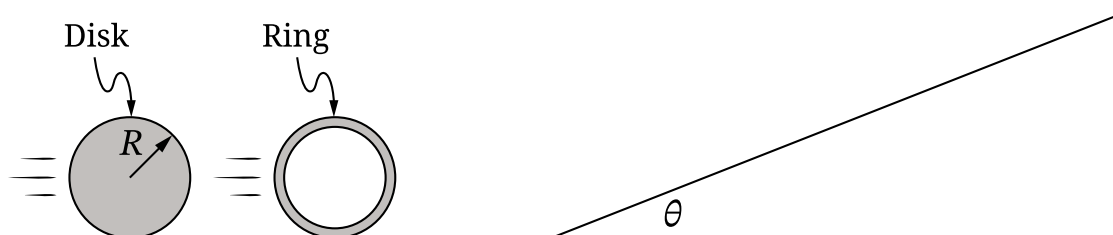


Figure 1

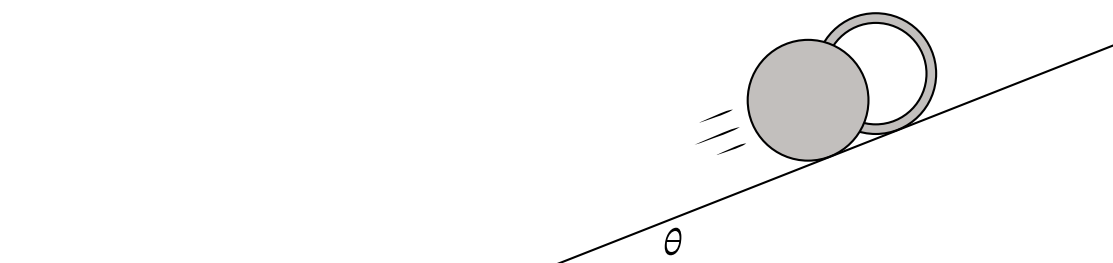


Figure 2

- A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R , respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

- B.** A cylinder has mass M , radius R , and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M , R , I , θ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C.** In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

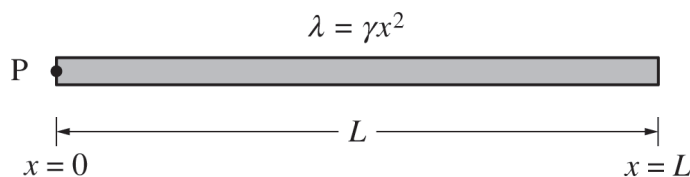
Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

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END OF EXAM

Begin your response to QUESTION 3 on this page.



3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \frac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L .

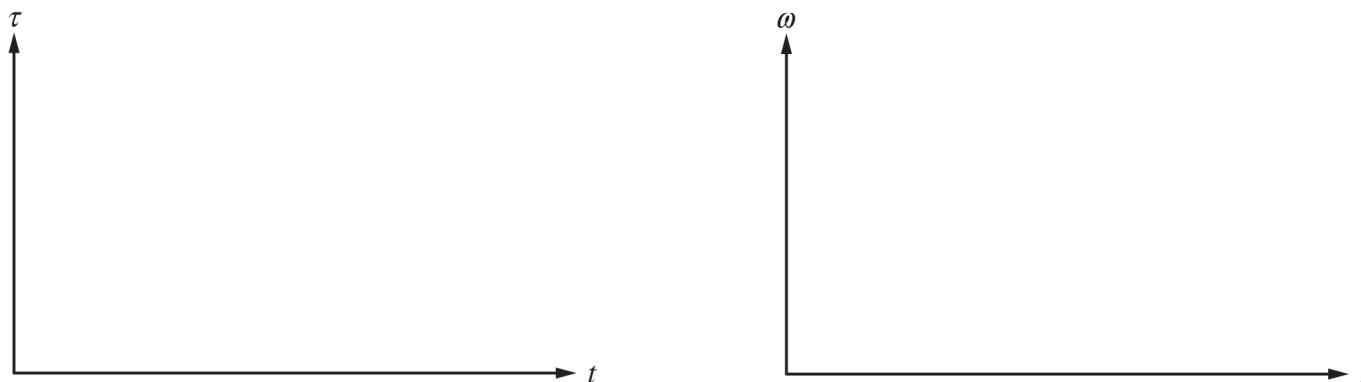
(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.



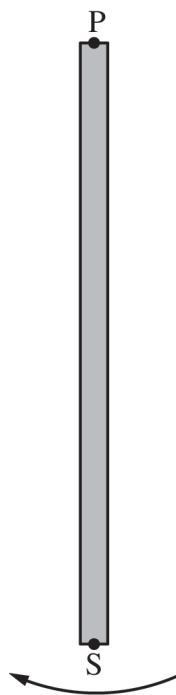
(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_____ Increasing _____ Decreasing _____ Not changing

Justify your answer.

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(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

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Name:

AP Physics C: **Mechanics: 2021 Set 1**

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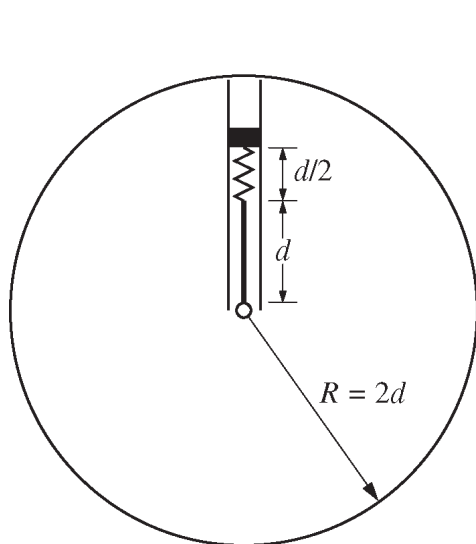


Figure 1

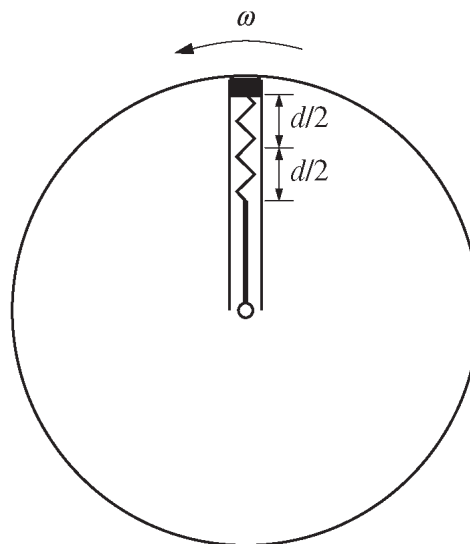


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

- (a) Derive an expression for the spring constant of the spring.
- (b)
 - i. Determine an expression for the rotational inertia of the block around the axis of the platform.
 - ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.
- (c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

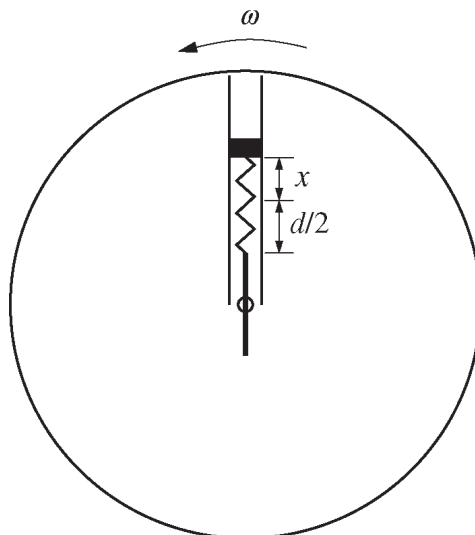


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

- (d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

- (e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

☐ Increasing ☐ Decreasing ☐ Staying the same

Justify your answer.

- (f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

☐ Positive ☐ Negative ☐ No work

Justify your answer.

- (g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

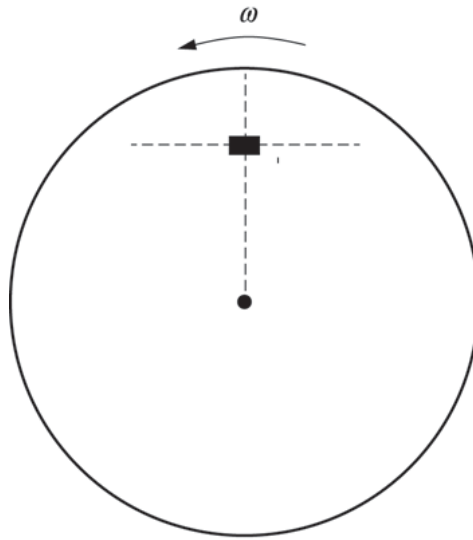


Figure 4

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END OF EXAM