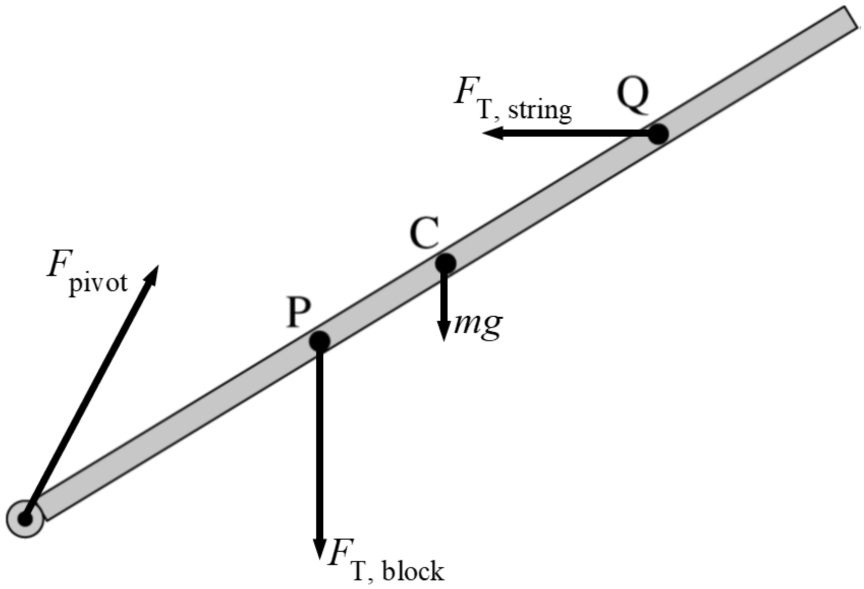
Question 3: Free-Response Question — 15 points

(a)	For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C **Scoring Note:** Labeling the downward force of tension as F_{block}, $3mg$, or similar, may earn this point.	**1 point**
	For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q	**1 point**
	For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present	**1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p, F_{pivot}, F_{n}, F_{N}, N, "normal force," "pivot force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_{T}, T, F_{string}, and "Force from string."

Total for part (a) 3 points

(b) For indicating that the net torque exerted on the rod is equal to zero **1 point**

Example Response

$\Sigma\tau = 0$

For a correct expression for the torque exerted on the rod by the hanging mass **1 point**

Example Response

$$3mg\sin\theta\left(\frac{3L}{8}\right)$$

For a correct expression for the torque exerted on the rod by the gravitational force **1 point**

Example Response

$$mg\sin\theta\left(\frac{4L}{8}\right)$$

For a correct expression for the torque exerted on the rod by the string **1 point**

Example Responses

$F_T\sin(90° - \theta)\left(\frac{6L}{8}\right)$ **OR** $F_T\cos\theta\left(\frac{6L}{8}\right)$

Example Solution

$$\Sigma\tau = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\sin(90° - \theta)\left(\frac{6L}{8}\right) = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$$

$$\left(\frac{9}{8}\right)mg\sin\theta + \left(\frac{4}{8}\right)mg\sin\theta = \left(\frac{6}{8}\right)F_T\cos\theta$$

$$13mg\sin\theta = 6F_T\cos\theta$$

$$F_T = \frac{13}{6}mg\tan\theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) 4 points

(c)	For indicating that the torque exerted on the rod by the string is always the same	**1 point**
	For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases	**1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T,\,\text{new}}$ *must decrease.*

Total for part (c) 2 points

(d)(i)	For indicating the total mass is the sum of differentiable masses along the length of the rod	**1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$M = 14.4$ kg

Example Solution

$$M = \int dm$$
$$M = \int \lambda\, dx$$
$$M = \int_0^{1.2} (6 + 10x)\, dx$$
$$M = \left(6x + \frac{10x^2}{2}\right)\Bigg|_0^{1.2\text{ m}}$$
$$M = 6\text{ kg/m}(1.2\text{ m}) + \frac{10\text{ kg/m}^2\,(1.2\text{ m})^2}{2}$$
$$M = 14.4\text{ kg}$$

(d)(ii) For a correct substitution of λ into an integral expression of rotational inertia **1 point**

Example Response

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \lambda dr \text{ and } r = x$$

$$I = \int_0^{1.2\text{ m}} \lambda x^2 dx$$

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

$$I = \frac{(6.0 \text{ kg/m})(1.2 \text{ m})^3}{3} + \frac{\left(10.0 \text{ kg/m}^2\right)(1.2 \text{ m})^4}{4}$$

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

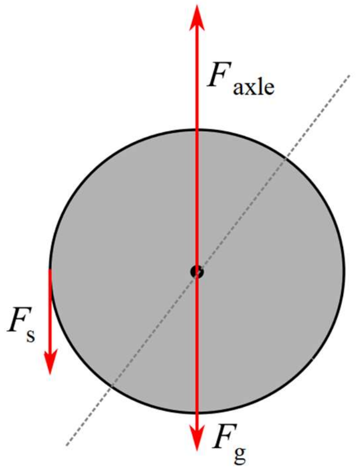
Total for part (d) 6 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a)	For indicating that the sum of the torques on the disk equals zero $\Sigma\tau_{\text{on disk}} = 0$ $\tau_g = \tau_s$ **OR** For indicating that the sum of the forces equals zero $\Sigma F = 0$ $F_g = F_s$	**1 point**
	For correctly substituting the expressions for the forces $F_g R = F_s R$ $m_B g R = k\Delta x R$ $m_B g = k\Delta x$ **OR** $F_g = F_s$ $m_B g = k\Delta x$	**1 point**
	For correctly substituting for Δx $m_B g = kR\theta$ $m_B = \dfrac{kR\theta}{g}$	**1 point**
	Total for part (a)	**3 points**
(b)	For drawing and labeling the force of the tension exerted on the disk anywhere between point P and the left edge of the disk, including point P and the left edge of the disk, tangent to the disk	**1 point**
	For correct location and label of the force due to gravity exerted on the disk, directed straight down	**1 point**
	For correct location and label of the force exerted on the disk by the axle, directed such that the disk remains in translational equilibrium (i.e., $\Sigma F = 0$)	**1 point**

Example Response



Scoring Notes:

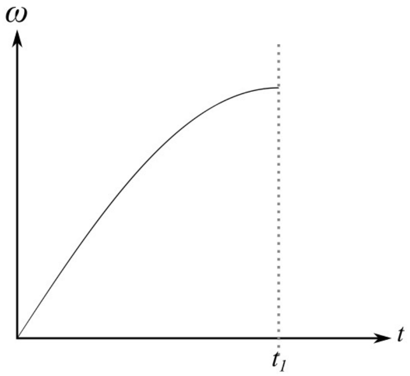
- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force, which can be used instead of F_{axle}. F_{spring}, F_s, T_{spring}, T, "spring force," or similar labels may be used for the tension force exerted by the spring.
- A response with extraneous forces or vectors can earn a maximum of two points.

	Total for part (b)	**3 points**
(c)	For indicating that the net torque is due only to the force exerted on the disk by the tension in the rotational form of Newton's second law $\tau_s = I_d\alpha$	**1 point**
	For correctly expressing the torque on the disk by the tension in terms of the spring force, which is equal to the tension, and the lever (moment) arm $F_sR = I_d\alpha$	**1 point**
	For correctly substituting for F_s $-k\Delta xR = I_d\alpha$	**1 point**
	For correctly substituting I_d and Δx, or an expression for Δx consistent with part (a) $-k(R\theta)R = \frac{1}{2}M_dR^2\alpha$ $\alpha = -\frac{2k\theta}{M_d}$ **Scoring Note:** The negative sign is not necessary to earn this point.	**1 point**
	Total for part (c)	**4 points**

(d) For a sketch that starts at zero and monotonically increases until time $t = t_1$ **1 point**

For a sketch that is concave down between time $t = 0$ and $t = t_1$ **1 point**

Example Response



Scoring Note: Any part of the graph beyond t_1 is not considered in scoring.

Total for part (d) 2 points

(e) For indicating that the torque exerted by the force due to gravity on the disk increased **1 point**

Example Response

The force due to gravity on the disk now has a non-zero lever arm and hence it exerts a larger torque on the disk.

For indicating that the torque exerted by the tension caused by the force due to gravity on the block increased **1 point**

Example Response

The force exerted by the right side of the string (from the block) on the disk has a longer lever arm, hence the torque it exerts is larger.

For indicating that the torque exerted by the tension increased **1 point**

Example Response

The counterclockwise torque due to the tension caused by the spring must increase to counteract the increase in clockwise torques due to the force due to gravity of the disk and tension caused by the force of gravity due to block to keep the disk in equilibrium.

Scoring Notes:

- A response that references the torque due to the force at the axle staying the same can earn all 3 points.
- A response that references the torque due to the force on the axle changing, or any additional torques can earn a maximum of 2 points.

Total for part (e) 3 points

Total for question 3 15 points

Question 3: Free-Response Question — 15 points

(a) For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting γx^2 into the above equation — **1 point**

$$I = \int x^2 \left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma \left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

Total for part (a) — 2 points

(b) For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting γx^2 into the numerator of the above equation — **1 point**

For correctly substituting M into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

Total for part (b) — 3 points

(c)	For selecting "Greater than" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (c)

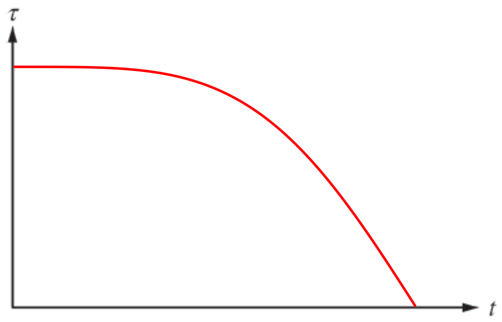
Because more of the mass of the rod is at the end of the rod opposite point P, more mass is concentrated away from the axis of rotation; thus, the rotational inertia of the rod would be greater around point P than around its center of mass.

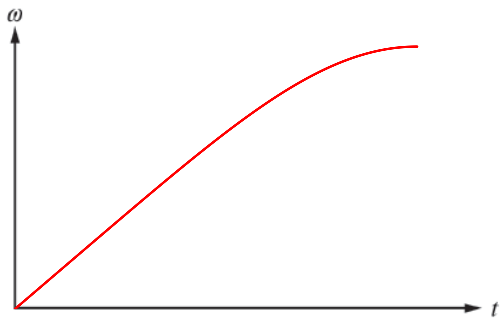
OR

According to the parallel axis theorem, $I = I_{cm} + md^2$, *if the axis is at a position away from the center of mass, the rotational inertia is larger than if the axis were at the center of mass.*

Total for part (c) 2 points

(d)	For a concave down curve that decreases to zero for the graph of τ as a function of t	**1 point**
	For a concave down curve that approaches horizontal for the graph of ω as a function of t	**1 point**
	For consistency between the two graphs	**1 point**





Total for part (d) 3 points

(e)	For selecting "Decreases" with an attempted justification	**1 point**
	For a correct justification	**1 point**

Example responses for part (e)

As the rod rotates downward, the angle θ *in the torque equation* $\tau = rF\sin\theta$ *decreases. Thus, the torque on the rod decreases.*

OR

As the rod rotates downward, the lever arm between point P and the rod's center of mass continues to decrease; thus, the torque on the rod decreases.

Total for part (e) 2 points

(f)	For using conservation of energy to calculate the speed of the rotating rod $U_i + K_i = U_f + K_f$ $U_i + 0 = 0 + K_f$ $U_i = K_f$	**1 point**
	For correctly substituting into the above equation $mgh_i = \frac{1}{2}I\omega_f^2$	**1 point**
	For correctly solving for the linear speed of point S $Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$ $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}\left(9.8 \text{ m/s}^2\right)(1.0 \text{ m})} = 4.9 \text{ m/s}$	**1 point**
	Total for part (f)	**3 points**
	Total for question 3	**15 points**