

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$ -direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$ -direction with constant speed v_0 , as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

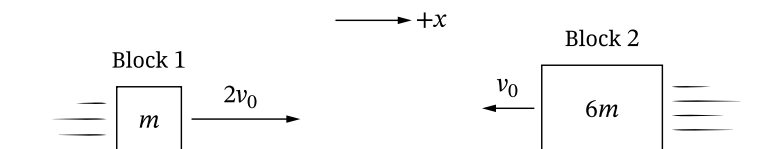


Figure 1

- A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

- i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.
- Arrows should start at the zero-momentum line.
 - The length of the arrows should be proportional to the relative magnitudes of the vectors.
 - Represent an arrow of zero length by drawing a dot at zero.

	Momentum Before Collision	Momentum After Collision
Block 1		
Block 2		
Two-Block System		

Figure 2

- ii. During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x -direction as a function of t that is modeled by $F(t) = F_{\max} \sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

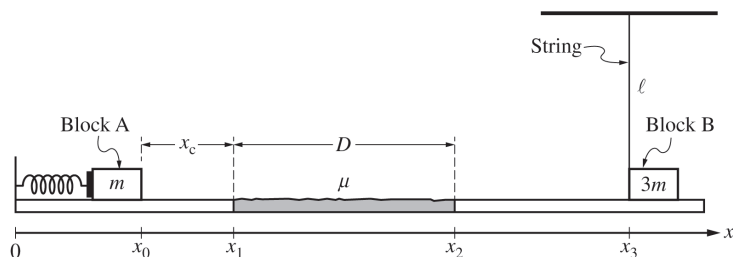
Derive an expression for $F_{\max}$. Express your answer in terms of m , v_0 , A , t_c , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- B. Consider a new scenario where Block 1 initially slides in the $+x$ -direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$ -direction with constant speed v_0 . The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0 . Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: MECHANICS****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D , where the coefficient of kinetic friction between Block A and the surface is μ . Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c .

GO ON TO THE NEXT PAGE.Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c . Block A moves with speed v , and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m , k , D , μ , x_c , and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1 .

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3 .

GO ON TO THE NEXT PAGE.

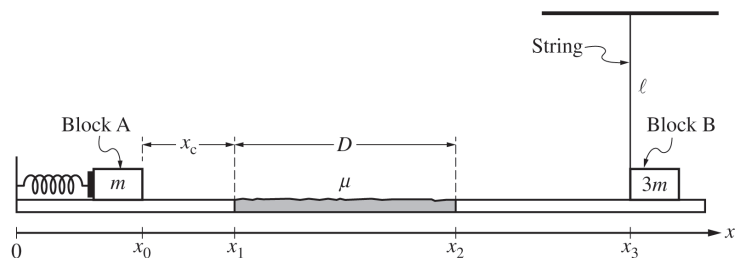
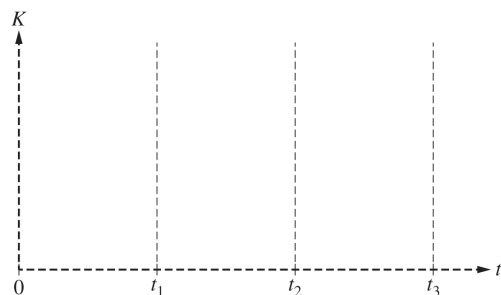
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 1

(b)

- i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3 .



- ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

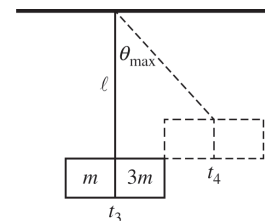
Continue your response to **QUESTION 1** on this page.Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4 , which occurs when the string makes a small angle $\theta_{\max}$ with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ . The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ .

- (c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

_____ $f_{2\ell} > f_\ell$ _____ $f_{2\ell} < f_\ell$ _____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\text{ m/s}^2$, $L = 0.25\text{ m/s}^4$, and t is the time in seconds. At $t = 6.0\text{ s}$, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

- (a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0\text{ s}$.
- (b) Calculate the speed of the rocket at $t = 6.0\text{ s}$.
- (c)
- i. Calculate the kinetic energy of the rocket at $t = 6.0\text{ s}$.
- ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0\text{ s}$.
- (d) Calculate the maximum height reached by the rocket relative to its launching point.
- (e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

