Question 2: Translation Between Representations (TBR) — 12 points

A	For indicating that K_{block} is zero	**Point A1**
	For drawing bars with positive heights for U_A and U_B	**Point A2**
	For drawing a bar for U_B with a height that is twice the height of the bar drawn for U_A **Scoring Note:** This point may be earned regardless of the signs of either bar.	**Point A3**

Example Response

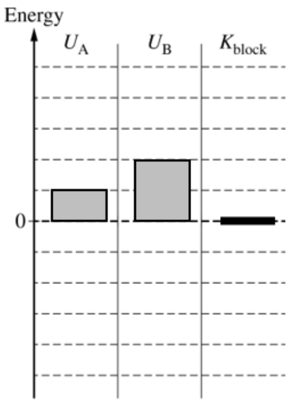


Figure 3

B	For a multistep derivation that includes energy conservation or simple harmonic motion	**Point B1**
	For relating the presence of both springs to the behavior of the system	**Point B2**
	For relating positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block	**Point B3**
	For a correct expression for v in terms of given quantities	**Point B4**

Example Responses

$$E_{\text{tot},0} = U_A(x_1) + U_B(x_1) + K_{\text{block}}(x_1)$$
$$E_{\text{tot},f} = U_A\left(\frac{1}{2}x_1\right) + U_B\left(\frac{1}{2}x_1\right) + K_{\text{block}}\left(\frac{1}{2}x_1\right)$$

$$E_{\text{tot},0} = \frac{1}{2}(k)(x_1)^2 + \frac{1}{2}(2k)(x_1)^2 + 0$$
$$E_{\text{tot},f} = \frac{1}{2}(k)\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{\text{block},\,\frac{1}{2}x_1}$$

$$E_{\text{tot},0} = E_{\text{tot},f}$$
$$\frac{1}{2}kx_1^2 + kx_1^2 = \frac{1}{8}kx_1^2 + \frac{1}{4}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$$
$$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{\text{block},\,\frac{1}{2}x_1}$$

$$K_{\text{block},\,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2$$
$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

OR

$$x = x_{\max}\cos(\omega t + \phi)$$

$$x(t) = x_1\cos\omega t$$
$$v(t) = -x_1\omega\sin\omega t$$

$$k_{\text{eff}} = k + 2k = 3k$$
$$\omega = \sqrt{\frac{3k}{m}}$$

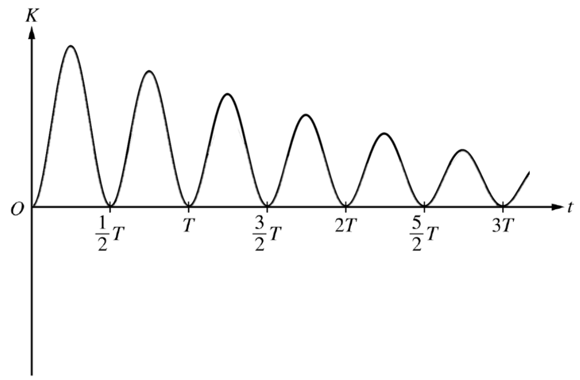
$$\frac{x_1}{2} = x_1\cos\omega t_{\frac{1}{2}x_1}$$
$$\cos\omega t_{\frac{1}{2}x_1} = \frac{1}{2}$$
$$t_{\frac{1}{2}x_1} = \frac{\pi}{3\omega}$$

$$v_{\frac{1}{2}x_1} = -x_1\omega\sin\omega t_{\frac{1}{2}x_1}$$
$$v_{\frac{1}{2}x_1} = -x_1\sqrt{\frac{3k}{m}}\sin\frac{\pi}{3}$$

$$v = \left|v_{\frac{1}{2}x_1}\right| = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

C	For sketching a curve that starts at zero and is always positive or zero	**Point C1**
	For sketching a periodic curve with zeros that have a period of $\frac{1}{2}T$	**Point C2**
	For sketching a periodic curve with a decreasing amplitude	**Point C3**

Example Response



Scenario 2

Figure 5

D	For indicating **one** of the following: • The period increases. • The rate at which the graph approaches zero increases. • The maximum value of K over each period is less than the graph drawn.	**Point D1**
	For a correct justification relevant to the feature indicated	**Point D2**

Example Responses

The period of the graph increases. The period of a simple harmonic oscillator increases with increasing mass.

OR

The rate at which the graph approaches zero amplitude increases. Because the mass of the block is larger, the force of friction is greater, which causes the block to lose energy at a greater rate.

Question 1: Free-Response Question — 15 points

(a)(i)	For a multi-step derivation with an application of the conservation of mechanical energy that indicates that all of the energy of the system is initially U_s	**1 point**

Example Response

$$E_{\text{initial}} = E_{\text{final}}$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$

For a correct solution for v — **1 point**

Example Response

$$v = x_c\sqrt{\frac{k}{m}}$$

Example Solution

$$E_{\text{initial}} = E_{\text{final}}$$
$$U_s = K$$
$$\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{kx_c^2}{m}}$$
$$v = x_c\sqrt{\frac{k}{m}}$$

(a)(ii) For a derivation to solve for the speed at x_2 that includes **one** of the following: **1 point**

- An appropriate application of the conservation of energy
- An appropriate kinematics equation

Example Responses

$K_{\text{initial}} - \Delta E_{\text{friction}} = K_{\text{final}}$ **OR** $v^2 = v_0^2 + 2a\Delta x$

For **one** of the following that is consistent with the previous point in the response for part (a)(ii): **1 point**

- A correct expression for the energy dissipated by friction
- A correct expression for the acceleration of the block in the region with nonnegligible friction

Example Responses

$\Delta E_{\text{friction}} = \mu mgD$ **OR** $a = -\mu g$

For attempting to derive an expression for $v_{\text{A,B}}$ by using the conservation of momentum **1 point**

Example Response

$m_{\text{initial}} v_{\text{initial}} = m_{\text{A,B}} v_{\text{A,B}}$

For substituting the expression for the speed at x_2 that is consistent with the first point of the response in part (a)(ii) and substituting the correct masses into an expression for conservation of momentum **1 point**

Example Response

$$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$$

Example Solutions

$E_{x_1} = E_{\text{before collision}}$

$K_{x_1} - \Delta E_{\text{friction}} = K_{\text{before collision}}$

$\frac{1}{2}m\left(\sqrt{\frac{kx_c^2}{m}}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{\text{A,B}} v_{\text{A,B}}$

$m_A v_2 = (m + 3m) v_{\text{A,B}}$

$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$

$v_{\text{A,B}} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

OR

$v_2^2 = v^2 + 2a\Delta x$

$v_2^2 = \left(x_c\sqrt{\frac{k}{m}}\right)^2 + 2aD$

$v_2 = \sqrt{\frac{kx_c^2}{m} + 2aD}$

$\Sigma F_x = -F_f = ma$

$-\mu mg = ma$

$a = -\mu g$

$v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

$\Sigma p_{\text{before collision}} = \Sigma p_{\text{after collision}}$

$m_A v_2 = m_{\text{A,B}} v_{\text{A,B}}$

$m_A v_2 = (m + 3m) v_{\text{A,B}}$

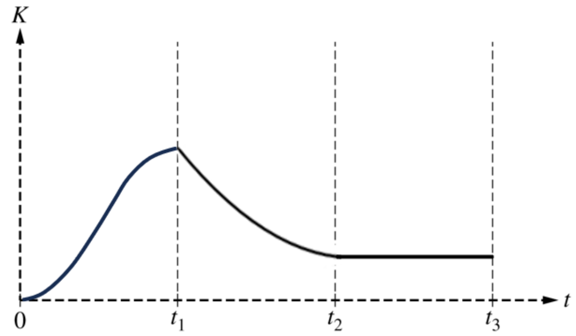
$v_{\text{A,B}} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m}$

$v_{\text{A,B}} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$

Total for part (a) 6 points

(b)(i)	For a nonlinear sketch that begins at zero and increases for the entire time interval $0 \le t \le t_1$	**1 point**
	For a sketch that decreases for the entire time interval $t_1 \le t \le t_2$ but does not go to zero	**1 point**
	For a sketch that is concave up for the time interval $t_1 \le t \le t_2$	**1 point**
	For a continuous function for the time interval $t_1 \le t \le t_3$ that has a horizontal line that is greater than zero for the time interval $t_2 \le t \le t_3$	**1 point**

Example Response



(b)(ii)	For a statement about the change in kinetic energy that is consistent with the graph drawn in the response for part (b)(i)	**1 point**
	For a correct explanation for why the kinetic energy is increasing, such as **one** of the following:	**1 point**

- An increasing graph means positive work is being done on the block.
- An external force is exerted on Block A, causing the velocity of the block to increase and the kinetic energy of the block to increase.
- Mechanical energy is conserved and/or there is no work done for the block-spring system, and the potential energy decreases.

For a correct explanation for why the graph is nonlinear, such as **one** of the following: **1 point**

- The rate at which the slope of the graph changes is related to the rate at which work is being done on the block.
- The external force exerted on Block A is changing, which causes a nonuniform change in the velocity of Block A, which results in a nonuniform change in kinetic energy.

Example Response

From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate, the kinetic energy of the block does not increase at a constant rate.

Total for part (b) 7 points

(c)	For selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification	**1 point**
	For correctly applying an equation that relates the length of a pendulum to the period or frequency of the pendulum	**1 point**

Example Response

The period of a pendulum is calculated by using $T = 2\pi\sqrt{\frac{l}{g}}$. Therefore, as the length is increased, the period will also increase. Because frequency and period are inversely related, an increase in period will result in a decrease in frequency.

Total for part (c) 2 points

Total for question 1 15 points

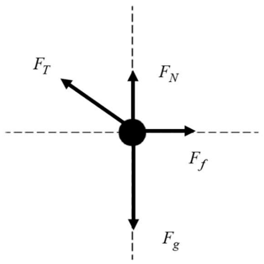
Question 1: Free-Response Question — 15 points

(a) For correctly drawing and labeling the force of gravity and the normal force on the block of mass m — **1 point**

For correctly drawing and labeling the force of friction on the block of mass m — **1 point**

For correctly drawing and labeling the force of tension on the block of mass m — **1 point**

Example Response



Scoring Notes:

- Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth" $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force. F_{string}, F_s, F_T, $F_{Tension}$, T, "string force," "tension force," or similar labels may be used for the tension force exerted by the string.
- A response with extraneous forces or vectors can earn a maximum of two points.

Total for part (a) — 3 points

(b) For a trigonometric expression relating the angle to the horizontal distance of the block from the left corner of the table, x. — **1 point**

For any correct trigonometric expression for θ in terms of the given quantities — **1 point**

Example Responses

First Point	Second Point
$\sin\theta = \dfrac{H}{\sqrt{H^2+x^2}}$	$\theta = \sin^{-1}\left(\dfrac{H}{\sqrt{H^2+x^2}}\right)$
$\cos\theta = \dfrac{x}{\sqrt{H^2+x^2}}$	$\theta = \cos^{-1}\left(\dfrac{x}{\sqrt{H^2+x^2}}\right)$
$\tan\theta = \dfrac{H}{x}$	$\theta = \tan^{-1}\left(\dfrac{H}{x}\right)$

Total for part (b) — 2 points

(c)(i) For using Newton's second law to sum the forces in the vertical direction and write an equation that is consistent with part (a) — **1 point**

$$\Sigma F_y = ma$$
$$F_{net,y} = F_N + F_{T,y} - F_g = 0$$
$$F_N = F_g - F_{T,y}$$

For correctly substituting the vertical component of the tension in terms of the given variables consistent with part (b) — **1 point**

$$F_N = mg - F_T\sin\theta$$
$$F_N = mg - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)$$

(c)(ii) For using Newton's second law to sum the forces in the horizontal direction and write an equation that is consistent with part (a) — **1 point**

$$F_{net,x} = F_{T,x} - F_f$$

For correctly substituting the horizontal component of the force of tension in terms of the given variables consistent with part (b) — **1 point**

$$F_{net,x} = F_T\cos\theta - F_f$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - F_f$$

For correctly substituting the expression for the normal force from part (c)(i) into the expression for the force of friction — **1 point**

$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - \mu_k F_N$$
$$F_{net,x} = F_T\left(\frac{(x)}{\sqrt{H^2+x^2}}\right) - \mu_k\left(mg - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)\right)$$

Total for part (c) — 5 points

(d) For any indication that the work done on the block by the string is due only to the horizontal component of the tension in the string — **1 point**

$$W = \int F_{T,x}dx$$

For using the horizontal component of the force of tension consistent with part (c) — **1 point**

For indicating that work is the integral of the force with respect to x, including limits or a constant of integration — **1 point**

$$W = \int_{x=L}^{x=0} -F_T\left(\frac{x}{\sqrt{H^2+x^2}}\right)dx$$

Total for part (d) — 3 points

(e)	For selecting "More work …" with an attempt at a relevant justification	**1 point**
	For a correct justification relating the smaller angle to a larger component of the force of tension, thus resulting in greater work.	**1 point**

Example Response

F_T stays the same for both halves, the displacement is the same in both halves, but from $x = L$ to $x = L/2$ the angle is smaller, resulting in a larger component of the tension force that aligns with the displacement.

Total for part (e) 2 points

Total for question 1 15 points