Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

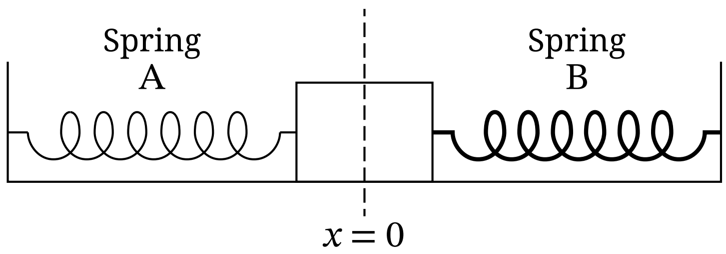


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

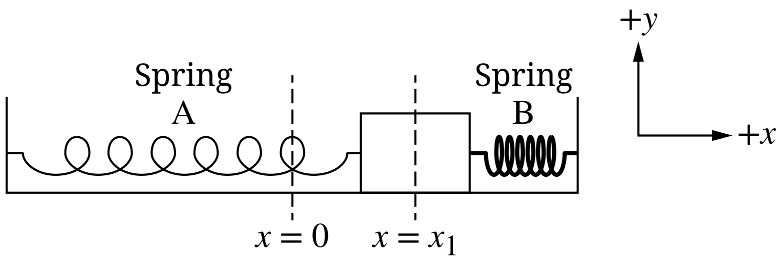


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

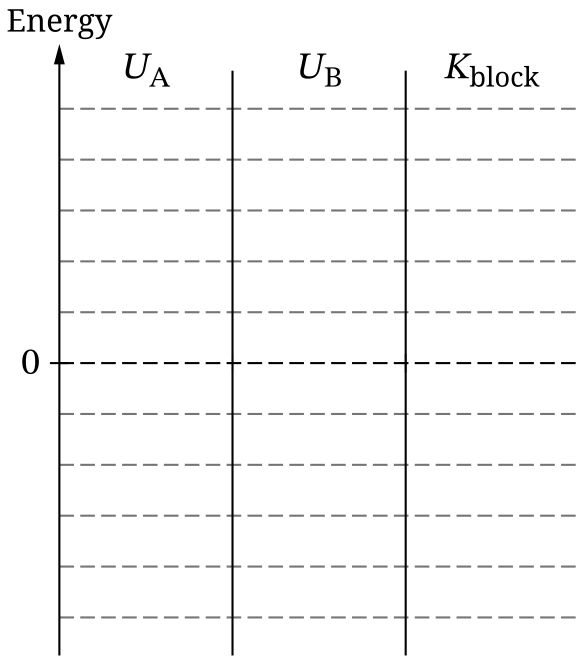
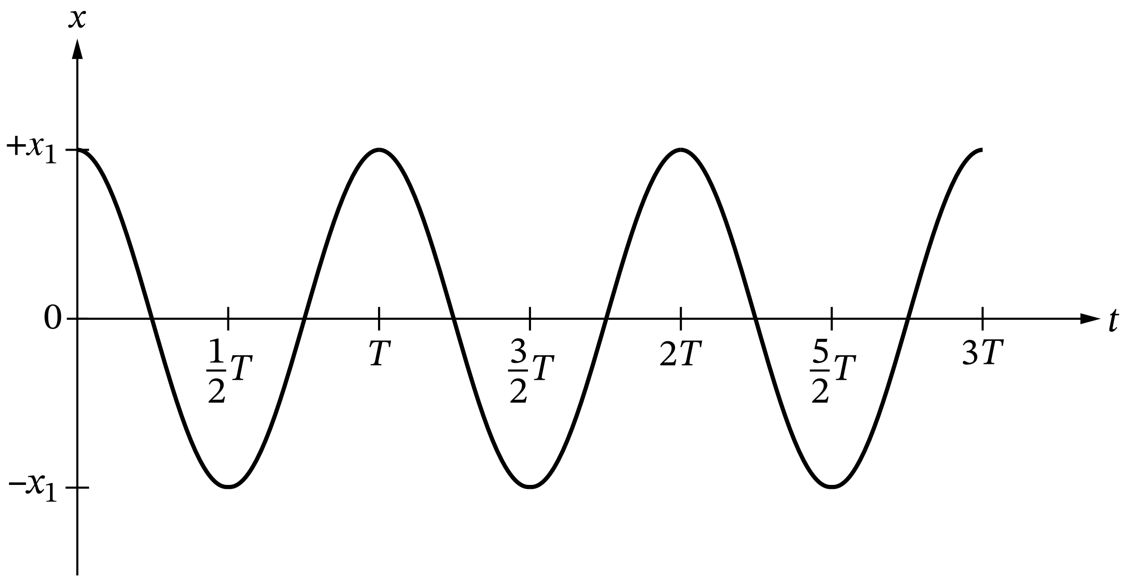


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

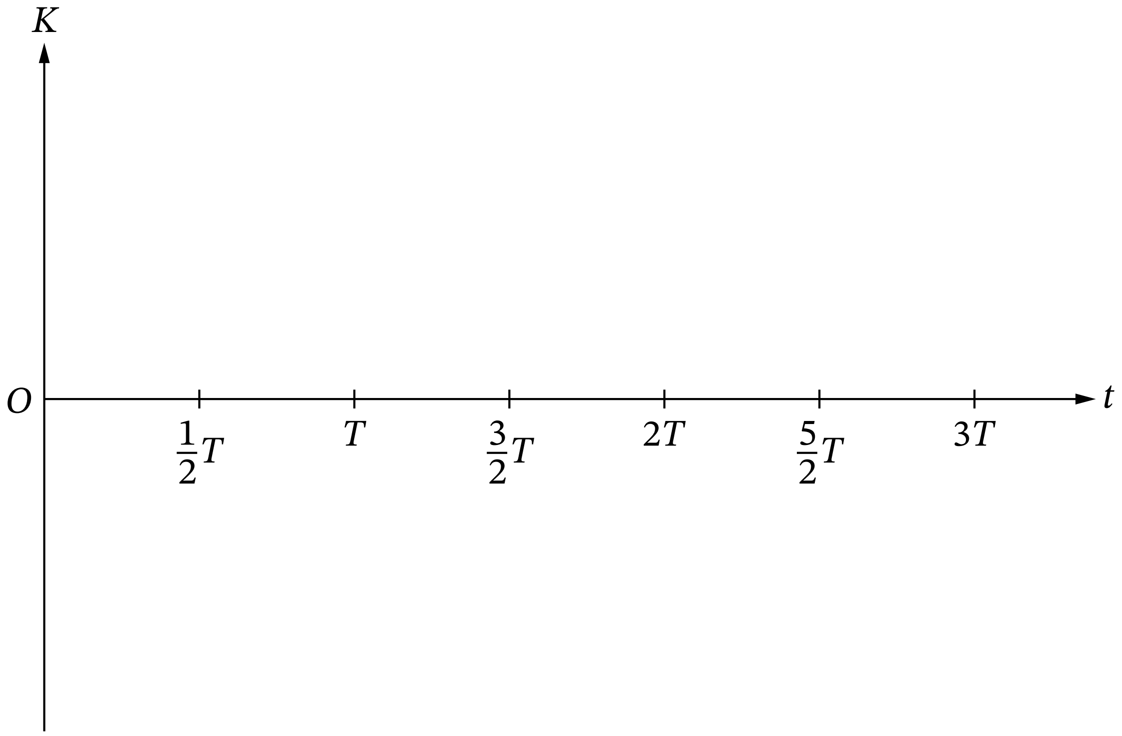


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Begin your response to **QUESTION 1** on this page.

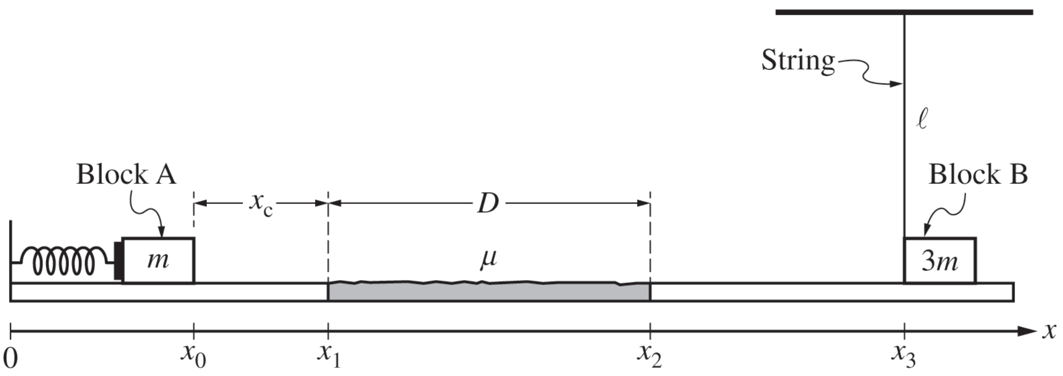
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

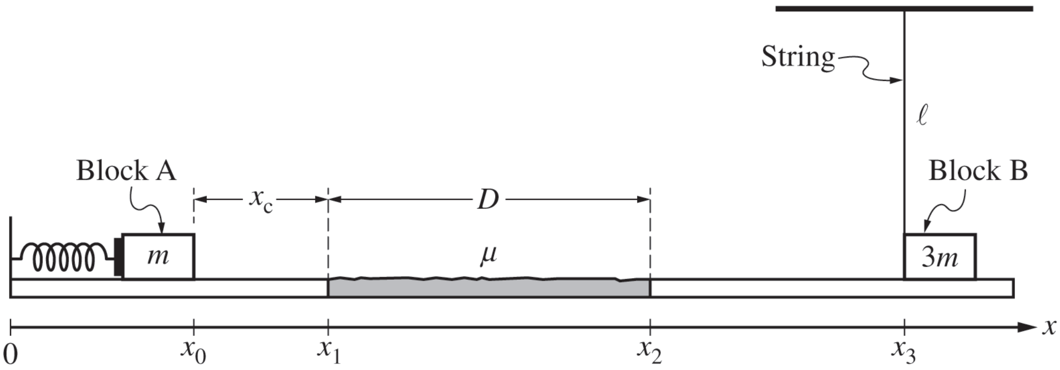
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

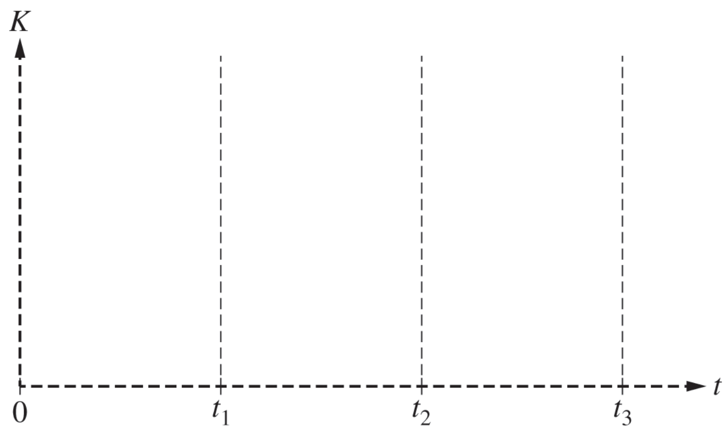


<u>Note:</u> Figure not drawn to scale.

Figure 1

(b)

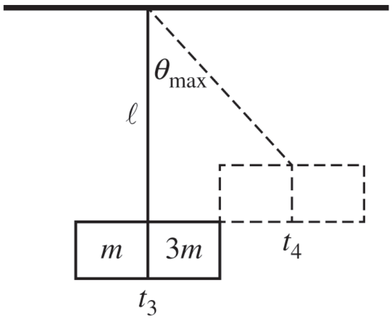
i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

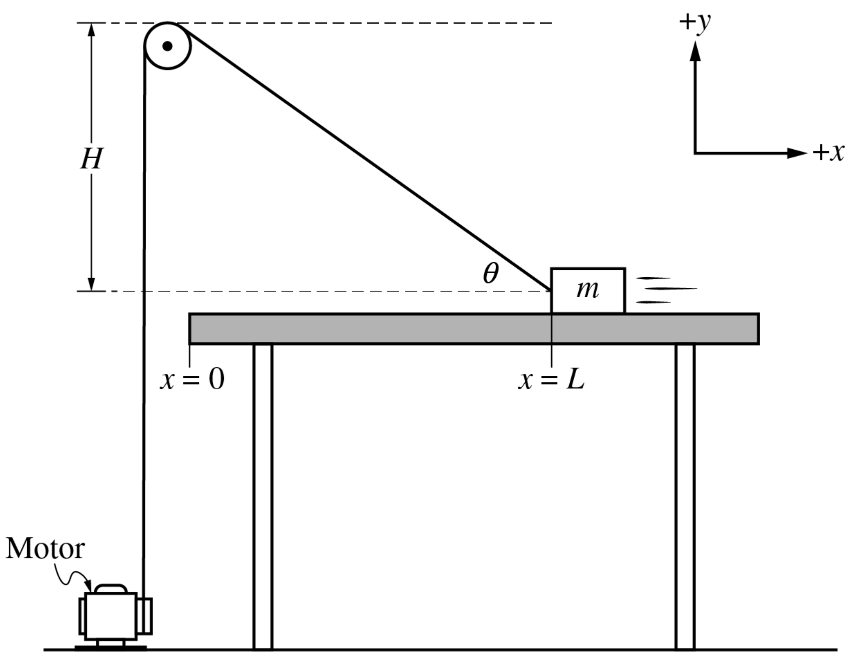
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

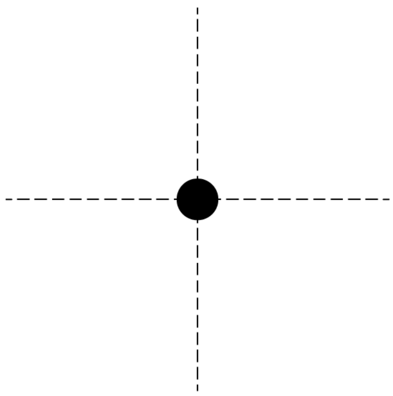


1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(c)

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

ii. Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.

(e) Does the string do more, less, or the same amount of work on the block as the block moves from $x = L$ to $x = \frac{L}{2}$ compared to when the block moves from $x = \frac{L}{2}$ to $x = 0$?

_______ More work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ Less work when the block moves from $x = L$ to $x = \frac{L}{2}$

_______ The same amount of work when the block moves from $x = L$ to $x = \frac{L}{2}$

Justify your answer.

GO ON TO THE NEXT PAGE.