

Question 3: Free-Response Question

15 points

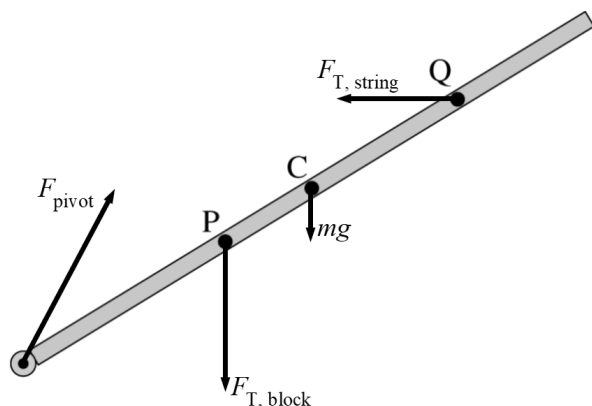
- (a) For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C **1 point**

Scoring Note: Labeling the downward force of tension as F_{block} , $3mg$, or similar, may earn this point.

For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q **1 point**

For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present **1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “F Earth on block,” “F on block by Earth,” $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p , F_{pivot} , F_n , F_N , N , “normal force,” “pivot force.”

Scoring Note: Examples of appropriate labels for the tension force include F_t , F_T , T ,

F_{string} , and “Force from string.”

Total for part (a) 3 points

- (b) For indicating that the net torque exerted on the rod is equal to zero **1 point**

Example Response

$$\Sigma \tau = 0$$

For a correct expression for the torque exerted on the rod by the hanging mass **1 point**

Example Response

$$3mg \sin \theta \left(\frac{3L}{8} \right)$$

For a correct expression for the torque exerted on the rod by the gravitational force **1 point**

Example Response

$$mg \sin \theta \left(\frac{4L}{8} \right)$$

For a correct expression for the torque exerted on the rod by the string **1 point**

Example Responses

$$F_T \sin(90^\circ - \theta) \left(\frac{6L}{8} \right) \quad \text{OR} \quad F_T \cos \theta \left(\frac{6L}{8} \right)$$

Example Solution

$$\Sigma \tau = 0$$

$$3mg \sin \theta \left(\frac{3L}{8} \right) + mg \sin \theta \left(\frac{4L}{8} \right) - F_T \sin(90^\circ - \theta) \left(\frac{6L}{8} \right) = 0$$

$$3mg \sin \theta \left(\frac{3L}{8} \right) + mg \sin \theta \left(\frac{4L}{8} \right) - F_T \cos \theta \left(\frac{6L}{8} \right) = 0$$

$$\left(\frac{9}{8} \right) mg \sin \theta + \left(\frac{4}{8} \right) mg \sin \theta = \left(\frac{6}{8} \right) F_T \cos \theta$$

$$13mg \sin \theta = 6F_T \cos \theta$$

$$F_T = \frac{13}{6} mg \tan \theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions (sin and cos) are reversed for all three torque terms.

Total for part (b) 4 points

- (c) For indicating that the torque exerted on the rod by the string is always the same **1 point**
- For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases **1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T, \text{new}}$ must decrease.

Total for part (c) 2 points

- (d)(i) For indicating the total mass is the sum of differentiable masses along the length of the rod **1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x) dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$$M = 14.4 \text{ kg}$$

Example Solution

$$M = \int dm$$

$$M = \int \lambda dx$$

$$M = \int_0^{1.2} (6 + 10x) dx$$

$$M = \left(6x + \frac{10x^2}{2} \right) \bigg|_0^{1.2 \text{ m}}$$

$$M = 6 \text{ kg/m}(1.2 \text{ m}) + \frac{10 \text{ kg/m}^2 (1.2 \text{ m})^2}{2}$$

$$M = 14.4 \text{ kg}$$

- (d)(ii) For a correct substitution of λ into an integral expression of rotational inertia **1 point**

Example Response

$$I = \int_0^{1.2 \text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \bigg|_0^{1.2 \text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \quad dm = \lambda dr \quad \text{and} \quad r = x$$

$$I = \int_0^{1.2 \text{ m}} \lambda x^2 dx$$

$$I = \int_0^{1.2 \text{ m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4} \right) \bigg|_0^{1.2 \text{ m}}$$

$$I = \frac{(6.0 \text{ kg/m})(1.2 \text{ m})^3}{3} + \frac{(10.0 \text{ kg/m}^2)(1.2 \text{ m})^4}{4}$$

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Total for part (d) 6 points**Total for question 3 15 points**

Question 3: Free-Response Question

15 points

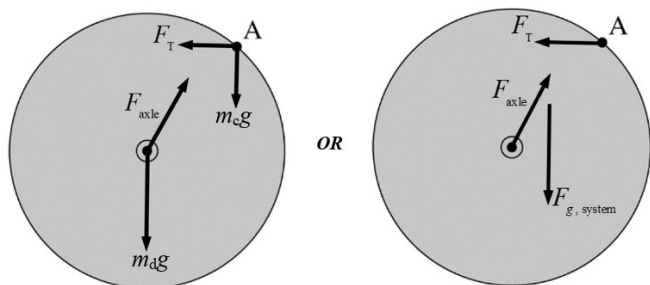
- (a) For drawing and appropriately labeling separate gravitational forces that are exerted on the disk and the lump of clay and exerted at the correct locations **1 point**

Scoring Note: Drawing a downward gravitational force exerted on the clay-disk system at the correct location can earn this point.

For drawing and appropriately labeling a leftward force exerted on the system at Point A **1 point**

For drawing and appropriately labeling a force directed up and right that is exerted on the system at the axle, and no extraneous forces are present **1 point**

Example Responses



Scoring Note: Examples of appropriate labels for the gravitational force include F_G , F_g , F_{grav} , W , mg , Mg , “grav force,” “ F Earth on disk,” “ F on disk by Earth,” $F_{\text{Earth on Disk}}$, $F_{\text{E,Disk}}$, and $F_{\text{Disk,E}}$. The labels G or g are not appropriate labels for the gravitational force.

Scoring Note: Examples of appropriate labels for the normal force from the axle include F_N , F_{axle} , N , “normal force,” and “axle force.”

Scoring Note: Examples of appropriate labels for the tension force include F_t , F_T , T ,

F_{string} , and “Force from string.”

Total for part (a) 3 points

- (b) For a multi-step derivation that indicates the net torque is zero **1 point**
- For indicating only the weight of the clay and the tension in the string exert torques on the system about the axle **1 point**

Example Response

$$\tau_{\text{net}} = \tau_{\text{clay}} - \tau_{\text{string}}$$

For a correct expression for the torque exerted on the system by the weight of the clay **1 point**

Example Response

$$\tau_{\text{clay}} = Rm_c g \cos \theta$$

For a correct expression for the torque exerted on the system by the tension in the string **1 point**

Example Response

$$\tau_{\text{string}} = RF_T \sin \theta$$

Example Solution

$$\Sigma \tau = 0$$

$$\tau_{\text{clay}} - \tau_{\text{string}} = 0$$

$$F_{g, \text{clay}} R \cos \theta - RF_T \sin \theta = 0$$

$$Rm_c g \cos \theta = RF_T \sin \theta$$

$$F_T = \frac{Rm_c g \cos \theta}{R \sin \theta}$$

$$F_T = m_c g \cot \theta$$

Scoring Note: A maximum of three points can be earned if the trigonometric functions (sin and cos) are reversed for both torque expressions.

Total for part (b) 4 points

(c) For indicating a direct relationship between the torque exerted by the clay and the tension in the string **1 point**

For correctly relating a greater torque exerted by the clay from at least **one** of the following: **1 point**

- An increase in the angle between the radial direction and the weight of the clay
- A greater perpendicular component of the weight of the clay
- A greater lever arm

Example Response

The torque caused by the weight of the clay at Point B is greater than when the clay is at Point A because the component of the weight that is perpendicular to the lever arm is larger. To maintain equilibrium, the net torque on the system is still zero, therefore the tension $F_{T, \text{new}}$ must be greater.

Total for part (c) 2 points

(d)(i) For using integration to calculate rotational inertia **1 point**

For **one** of the following: **1 point**

- Substituting $\rho(2\pi r)dr$ for dm
- Indicating the correct limits of integration

Example Response

$$I = \int_0^{0.3 \text{ m}} r^2 \rho(2\pi r) dr$$

For a correct answer of $I = 0.012 \text{ kg} \cdot \text{m}^2$, including units **1 point**

Example Response

$$I = 0.012 \text{ kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \quad dm = \rho dA \quad \text{and} \quad dA = 2\pi r dr$$

$$I = \int_0^R r^2 \rho dA$$

$$I = \int_0^{0.3 \text{ m}} 2\pi \rho r^4 dr$$

$$I = \frac{2\pi \rho R^5}{5} \bigg|_0^{0.3 \text{ m}}$$

$$I = \frac{2\pi(4.0 \text{ kg/m}^3)(0.3 \text{ m})^5}{5}$$

$$I = 0.012 \text{ kg} \cdot \text{m}^2$$

(d)(ii) For using the rotational form of Newton's second law **1 point**

For a correct expression for the net torque exerted on the clay-disk system **1 point**

Example Response

$$\tau_{\text{net}} = Rm_{\text{c}}g$$

For indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay **1 point**

Example Response

$$I_{\text{system}} = I_{\text{disk}} + I_{\text{clay}}$$

Example Solution

$$\Sigma \tau = I\alpha$$

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

$$\alpha = \frac{Rm_{\text{c}}g}{I_{\text{disk}} + I_{\text{clay}}}$$

$$\alpha = \frac{(0.3 \text{ m})(0.60 \text{ kg})(9.8 \text{ m/s}^2)}{\left((0.012 \text{ kg} \cdot \text{m}^2) + (0.60 \text{ kg})(0.3 \text{ m})^2\right)}$$

$$\alpha = 26.7 \text{ rad/s}^2$$

Total for part (d) 6 points

Total for question 3 15 points

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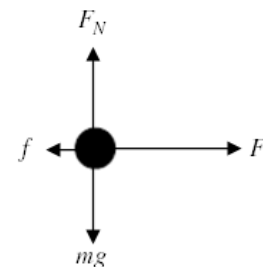
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2016 SCORING GUIDELINES

15 points total

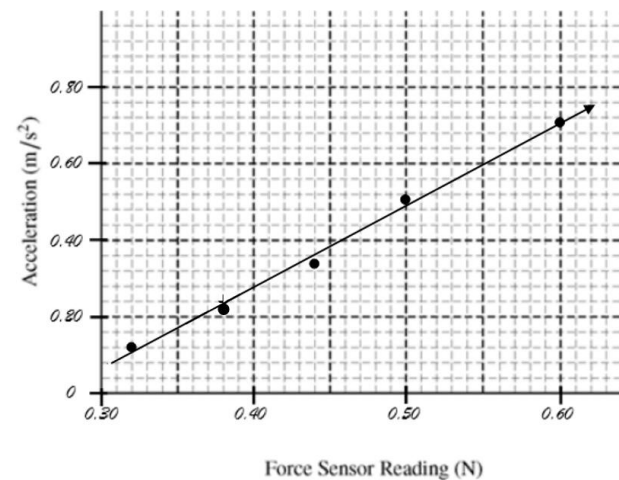
3 points

Question

Distribution
of points

For correctly drawing and labeling the force of tension 1 point
 For correctly drawing and labeling the force of friction 1 point
 For correctly drawing and labeling both forces in the vertical direction 1 point
 Note: A maximum of two points may be earned if there are any extraneous vectors.

3 points



For a correct scale that uses more than half the grid 1 point
 For correctly plotting the given data 1 point
 For drawing a straight line consistent with the given data 1 point
 Note: Full credit can be earned if the axes are switched.

SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

2 points

For correctly calculating slope using the best-fit straight line and not data points

1 point

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(0.70 - 0.16)}{(0.60 - 0.35)} \text{ kg}^{-1} = 2.16 \text{ kg}^{-1}$$

Note: Linear regression gives slope = 2.12 kg^{-1} .

For correctly calculating the mass of the cart using the slope

1 point

$$m = \frac{1}{\text{slope}} = \frac{1}{(2.16 \text{ kg}^{-1})}$$

Correct answer:

 $m = 0.463 \text{ kg}$ (Note: linear regression gives $m = 0.472 \text{ kg}$)

1 point

For an answer with correct units consistent with the x-intercept of the graph from (b) i.

1 point

$$f = 0.272 \text{ N}$$

1 point

Applying Newton's second law and substituting the values from part (b)

$$\sum F = ma = F_a - f$$

$$a = \frac{F_a - f}{m} = \frac{0.45 \text{ N} - 0.272 \text{ N}}{0.463 \text{ kg}}$$

For an answer with correct units consistent with part (b), either from the graph or calculated using the mass and frictional force.

1 point

$$a = 0.376 \text{ m/s}^2$$

SCORING GUIDELINES

Question 1 (continued)

Distribution
of points

3 points

For using a correct equation to solve for the speed of the cart when the string breaks, with an acceleration consistent with part (c) i.

1 point

$$v_2 = v_1 + at$$

$$v_2 = 0 + (0.376 \text{ m/s}^2)(2.0 \text{ s})$$

$$v_2 = 0.752 \text{ m/s}$$

For recognizing that the acceleration of the cart after the string breaks is due to the frictional force determined in part (b)

1 point

Use a correct equation to solve for the time for the cart to stop after the string breaks

$$v_2 = v_1 + at$$

$$0 = v_1 - \frac{f}{m}t$$

For using the final velocity before the string breaks as the initial velocity for the cart stopping in the correct equation for time

1 point

$$t = \frac{mv_1}{f}$$

$$t = \frac{(0.463 \text{ kg})(0.752 \text{ m/s})}{(0.272 \text{ N})}$$

Correct answer

$$t = 1.28 \text{ s}$$

Alternate solution

Alternate points

For setting the magnitude of the impulse before the string breaks equal to the magnitude of the impulse after the string breaks

1 point

$$F_1 t_1 = F_2 t_2$$

For correctly using the proper force (e.g., the tension force minus the friction force) for F_1

1 point

For correctly using the proper force (e.g., the friction force) for F_2

1 point

Correct answer

$$t = 1.28 \text{ s}$$

1 point

For selecting "Equal to"

1 point

1 point

For selecting "Greater than"

1 point

SCORING GUIDELINES

15 points total	Question	Distribution of points
2 points		
	For correctly drawing and labeling vectors on the block of mass $2M$	1 point
	For correctly drawing and labeling vectors on the block of mass $3M$ using symbols for the vectors that are physically correct and different from those on the block of mass $2M$	1 point
	Note: A maximum of one point can be earned if there are any extraneous vectors.	
2 points		
	Using a proper expression for conservation of momentum	
	$p_1 = p_2$	
	For correctly substituting into the above equation	1 point
	$3Mv_0 = (3M + 2M)v_f$	
	For a correct answer	1 point
	$v_f = \frac{3}{5}v_0$	
(1 point		
	Using a proper expression for kinetic energy of the two-block system	
	$K = \frac{1}{2}mv^2$	
	$K = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2$	
	For an answer consistent with part (b)	1 point
	$K = \frac{9}{10}Mv_0^2$	

SCORING GUIDELINES

Question continued)	Distribution of points
4 points	
For a correct expression of the conservation of energy	1 point
$\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$	
$K_0 = U_{\text{final}}$	
For attempting to integrate the spring force equation	1 point
$\frac{9}{10}Mv_0^2 = -\int_{x_1}^{x_2} -Bx^3 dx$	
$\frac{9}{10}Mv_0^2 = \int_{x_1}^{x_2} Bx^3 dx$	
For using the correct limits of integration or an appropriate constant of integration	1 point
$\frac{9}{10}Mv_0^2 = \int_0^D Bx^3 dx$	
$\frac{9}{10}Mv_0^2 = \left[\frac{Bx^4}{4}\right]_0^D$	
For an answer consistent with the speed from (b) or the kinetic energy from part (c)	1 point
$D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$	
2 points	
For selecting the correct answer "Right" with a reasonable attempt at a justification	1 point
If the incorrect selection is made, no points are earned for the justification.	
For an indication that at maximum compression the block of mass $2M$ has an acceleration to the right due to the forces acting on the block of mass $2M$ or an acceleration to the right due to the external spring force acting on the system of blocks	1 point
Example: At maximum compression the two-block system is instantaneously at rest. The only horizontal external force acting on the system is due to the spring. This force is directed to the right. The system and therefore the block of mass $2M$ is accelerated to the right, which implies that the net force acting on the block of mass $2M$ is also to the right.	

SCORING GUIDELINES

Question continued)

Distribution
of points

2 points

the magnitude of the net force is greater on the block of mass $3M$.

If the incorrect selection is made, no points are earned for the justification.

For an indication that both blocks will have the same acceleration

1 point

For a correct justification for why the net force is greater on the block of mass $3M$

1 point

Example:

Because the blocks stick together, both blocks must have the same acceleration.

Because the block of mass $3M$ has more mass, the net force on it must be greater than the net force on the block of mass $2M$.

2 points

For selecting the correct answer “No,” with a reasonable attempt at a justification

1 point

If the incorrect selection is made, no points are earned for the justification.

If the correct answer is selected without any justification, the point is not earned for the selection.

For an indication that the spring does not apply a linear force and that simple harmonic motion is the resulting motion of a linear restoring force

1 point

$$\left(a = -\frac{k}{m}\Delta x\right)$$

Example:

Because the blocks are sticking together and are attached to the spring, the spring will apply a restoring force to the blocks. However, because the restoring force exerted by a nonlinear spring is not proportional to the blocks' displacement from equilibrium, the blocks do not exhibit simple harmonic motion.

2016 SCORING GUIDELINES

Question

Distribution
of points

15 points total

3 points

For an indication that the spring force equals the mass times the centripetal acceleration

1 point

$$F_s = F_c = ma_c$$

$$kx = mr\omega^2$$

correct substitution for x in the equation

1 point

For a correct substitution for r in the equation that can be used to solve for k

1 point

$$k\left(\frac{d}{2}\right) = m(2d)\omega^2$$

correct answer:

$$k = 4m\omega^2$$

1 point

Substitute values for the block into the equation for rotational inertia

$$I = \sum mr^2 = m(2d)^2$$

For a correct answer or an answer consistent with the radius used in part (a)

1 point

$$I = 4md^2$$

2 points

For an indication that the rotational inertia of the system must include the platform, the rod, and the object

1 point

$$I = I_p + I_R + I_O$$

$$I = \frac{m_p R^2}{2} + \frac{m_R d^2}{3} + mr^2$$

For correctly substituting into the equation the rotational inertia of the platform, the rod, and the object consistent with (b) i.

1 point

$$I = \frac{5m(2d)^2}{2} + \frac{3m d^2}{3} + 4md^2$$

$$I = (10 + 1 + 4)md^2$$

correct answer:

$$I = 15md^2$$

1 point

Using a correct expression of angular momentum

$$L = I\omega$$

For an answer consistent with the answer from part (b) ii.

1 point

$$L = 15md^2\omega$$

SCORING GUIDELINES

Question continued)

Distribution
of points

3 points

For indicating that the spring force equals the mass times the centripetal acceleration

1 point

$$F_s = F_c = ma_c$$

$$kx = mr\omega^2$$

For a correct substitution for k , or a substitution consistent with part (a), in the equation

1 point

For a correct substitution for r in the equation that can be used to solve for x

1 point

$$(4m\omega^2)x = m(d + x)\omega^2$$

$$4x = d + x$$

$$3x = d$$

Correct answer:

$$x = d/3$$

2 points

selecting “Decreasing”

1 point

the wrong selection is made, no points are earned for the justification.

For a correct justification

1 point

Example: The rotational inertia I of the system decreases while the angular velocity ω stays the same. Because the angular momentum is $I\omega$, it decreases.

2 points

For selecting “Negative”

1 point

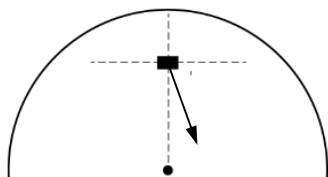
If the wrong selection is made, no points are earned for the justification.

For a correct justification

1 point

Example: Because the angular velocity is constant and the rotational inertia has decreased, the rotational kinetic energy has decreased. Therefore, work must be negative.

1 point



n arrow starting on the box and pointing down and to the right

1 point