Oscillations

AP Physics C: Mechanics ▪ Unit 7

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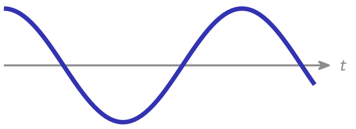
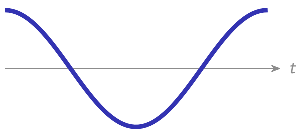


What we will cover



1 Defining SHM

SHM is a differential equation

A **restoring force** 回复力 $F = -kx$ put into $\vec{F} = m\vec{a}$ gives the defining law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \qquad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration is $-\omega^2$ times its displacement oscillates in SHM.

Foucault pendulum

2 Period

Frequency and period

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

The period does not depend on amplitude – a bigger swing just moves faster.

$0.5\,\mathrm{kg}$ on $k = 50\,\mathrm{N/m}$:

$$T = 2\pi\sqrt{\frac{0.5}{50}} \approx 0.63\,\mathrm{s}$$

Pull it 2 or $10\,\mathrm{cm}$ – same period. Heavier mass or softer spring lengthens T.

3 Analyzing SHM

Representing and analyzing SHM

The solution is a cosine; differentiate for v and a:

Position, velocity, acceleration

$$x = A\cos(\omega t + \phi)$$
$$v = -A\omega\sin(\omega t + \phi)$$
$$a = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x$$

$v_{max} = A\omega$ (at the middle);
$a_{max} = A\omega^2$ (at the extremes).

$A = 0.1$, $\omega = 5$: $v_{max} = 0.5\,\mathrm{m/s}$, $a_{max} = 2.5\,\mathrm{m/s^2}$. Velocity peaks a quarter-cycle before position.

4 Energy

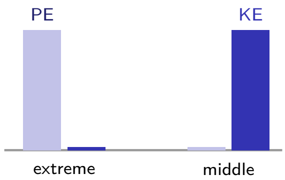
Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

Set by the amplitude

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

All **potential** 弹性势能 at the extremes; all **kinetic** 动能 in the middle – solve for the speed at any x.



$k = 200$, $A = 0.1$: $E = 1\,\mathrm{J}$ always.

Oscillations
└ Energy

Energy of an oscillator

Oscillations
└ Energy

Energy of an oscillator

5 Pendulums

Simple and physical pendulums

Simple pendulum (small angle)

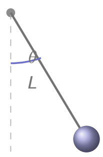
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

d = pivot to centre of mass.



$1\,\mathrm{m}$ pendulum: $T \approx 2.0\,\mathrm{s}$.

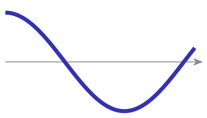
6 Recap

Unit 7 – key takeaways

1. **SHM**
   $d^2x/dt^2 = -\omega^2 x$ with $\omega = \sqrt{k/m}$
2. **Motion**
   $x = A\cos(\omega t + \phi)$; $v_{max} = A\omega$, $a_{max} = A\omega^2$
3. **Energy**
   $E = \frac{1}{2}kA^2$; trades between PE and KE
4. **Pendulums**
   $2\pi\sqrt{L/g}$ (simple), $2\pi\sqrt{I/mgd}$ (physical)

Check yourself

1. What differential equation defines simple harmonic motion?
2. Does a mass-spring period depend on the amplitude?
3. Where in the swing is the speed greatest?
4. Does a simple pendulum's period depend on the bob's mass?



Answers **1.** $d^2x/dt^2 = -\omega^2 x$ **2.** no **3.** in the middle **4.** no