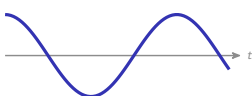


Oscillations

AP Physics C: Mechanics • Unit 7

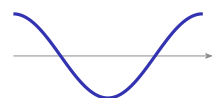
AP Physics C: Mechanics



Oscillations

What we will cover

- 1 Defining SHM as a differential equation
- 2 Frequency and period
- 3 Representing and analyzing SHM
- 4 Energy of an oscillator
- 5 Simple and physical pendulums



1 Defining SHM

Oscillations
└ Defining SHM

SHM is a differential equation

A **restoring force** 回复力 $F = -kx$ put into $\vec{F} = m\vec{a}$ gives the defining law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \quad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration is $-\omega^2$ times its displacement oscillates in SHM.



Foucault pendulum

2 Period

Oscillations
└ Period

Frequency and period

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}}$$

The period does **not** depend on amplitude – a bigger swing just moves faster.

0.5 kg on $k = 50 \text{ N/m}$:

$$T = 2\pi \sqrt{\frac{0.5}{50}} \approx 0.63 \text{ s}$$

Pull it 2 or 10 cm – same period. Heavier mass or softer spring lengthens T .

3 Analyzing SHM

Representing and analyzing SHM

The solution is a cosine; differentiate for v and a :

Position, velocity, acceleration

$$x = A \cos(\omega t + \phi)$$

$$v = -A\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$$

$$v_{\max} = A\omega \text{ (at the middle);}$$
$$a_{\max} = A\omega^2 \text{ (at the extremes).}$$

$A = 0.1$, $\omega = 5$: $v_{\max} = 0.5 \text{ m/s}$,
 $a_{\max} = 2.5 \text{ m/s}^2$. Velocity peaks a
quarter-cycle before position.

4 Energy

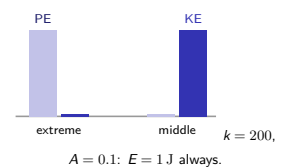
Energy of an oscillator

Energy sloshes between two forms, but the
total is constant:

Set by the amplitude

$$E = \frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

All **potential** 弹性势能 at the ex-
tremes; all **kinetic** 动能 in the middle
— solve for the speed at any x .



Energy of an oscillator

Energy of an oscillator

5 Pendulums

Oscillations
└ Pendulums

Simple and physical pendulums

Simple pendulum (small angle)

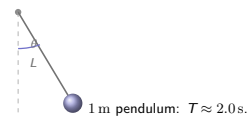
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

d = pivot to centre of mass.



6 Recap

Oscillations
└ Recap

Unit 7 – key takeaways

1 SHM

$$d^2x/dt^2 = -\omega^2x \text{ with } \omega = \sqrt{k/m}$$

2 Motion

$$x = A\cos(\omega t + \phi); v_{\max} = A\omega, a_{\max} = A\omega^2$$

3 Energy

$$E = \frac{1}{2}kA^2; \text{ trades between PE and KE}$$

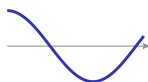
4 Pendulums

$$2\pi\sqrt{L/g} \text{ (simple), } 2\pi\sqrt{I/mgd} \text{ (physical)}$$

Oscillations
└ Recap

Check yourself

- 1 What differential equation defines simple harmonic motion?
- 2 Does a mass-spring period depend on the amplitude?
- 3 Where in the swing is the speed greatest?
- 4 Does a simple pendulum's period depend on the bob's mass?



Answers 1. $d^2x/dt^2 = -\omega^2x$ 2. no 3. in the middle 4. no