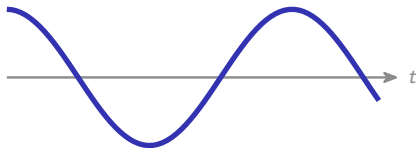


Oscillations

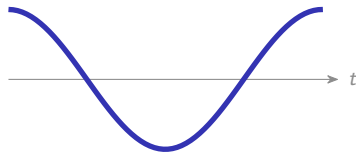
AP Physics C: Mechanics ■ Unit 7

AP Physics C: Mechanics



What we will cover

- 1 Defining SHM as a differential equation
- 2 Frequency and period
- 3 Representing and analyzing SHM
- 4 Energy of an oscillator
- 5 Simple and physical pendulums



1

Defining SHM

SHM is a differential equation

A **restoring force** 回复力 $F = -kx$
put into $\vec{F} = m\vec{a}$ gives the defining
law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \quad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration
is $-\omega^2$ times its displacement
oscillates in SHM.



For small angles a pendulum swings in SHM

What makes a motion simple harmonic

periodic motion 周期运动

anything that repeats; SHM is one special case of it

condition 1

there is an **equilibrium** 平衡位置 position the system returns to

condition 2

the restoring force is **proportional to the displacement** from it:
 $F = -kx$

the consequence

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$ – and every solution of that is a sinusoid

the payoff

the period is **independent of amplitude**: $T = 2\pi\sqrt{m/k}$

Whenever you can show a restoring force is proportional to displacement, you have proved the motion is SHM and may quote T immediately – that is the marked step.

2

Period

Frequency and period

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

The period does **not** depend on amplitude – a bigger swing just moves faster.

0.5 kg on $k = 50 \text{ N/m}$:

$$T = 2\pi\sqrt{\frac{0.5}{50}} \approx 0.63 \text{ s}$$

Pull it 2 or 10 cm – same period. Heavier mass or softer spring lengthens T . Each system also **ignores** one obvious variable: a pendulum's period does not depend on its mass, and a spring's does not depend on g .

3

Analyzing SHM

Representing and analyzing SHM

The solution is a cosine; differentiate for v and a :

Position, velocity, acceleration

$$x = A \cos(\omega t + \phi)$$

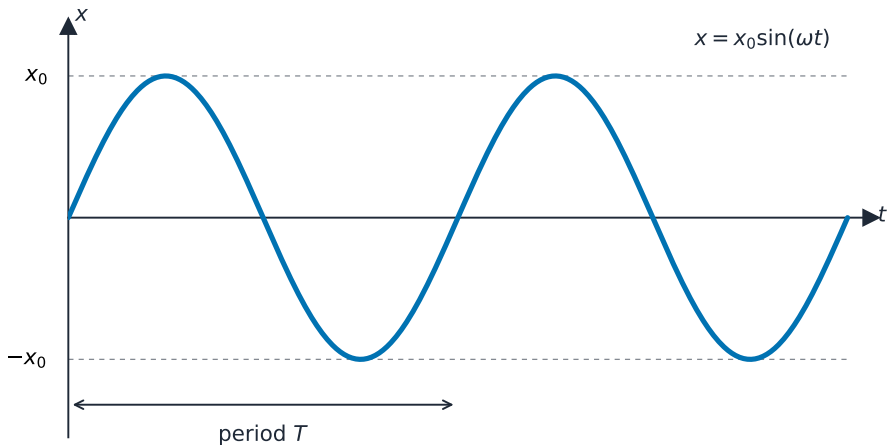
$$v = -A\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$$

$$v_{\max} = A\omega \text{ (at the middle);}$$
$$a_{\max} = A\omega^2 \text{ (at the extremes).}$$

$A = 0.1$, $\omega = 5$: $v_{\max} = 0.5 \text{ m/s}$,
 $a_{\max} = 2.5 \text{ m/s}^2$. Velocity peaks a
quarter-cycle before position.

SHM: displacement is sinusoidal



Displacement varies sinusoidally with time in simple harmonic motion

Where the speed is greatest, and where the acceleration is

at the ends $x = \pm A$

speed is **zero**, acceleration is **maximum** ($a_{\max} = \omega^2 A$) and points back to centre

at $x = 0$

acceleration is **zero**, speed is **maximum** ($v_{\max} = \omega A$)

the phase

x , v and a are each a quarter cycle apart – v leads x by 90° , a is 180° out of phase with x

the energy

$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ – constant, traded back and forth twice per cycle

The two extremes are never at the same place. If your answer has maximum speed *and* maximum acceleration at one point, a sign has gone astray.

4

Energy

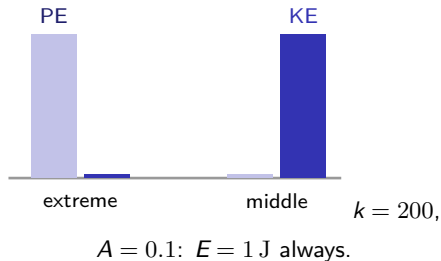
Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

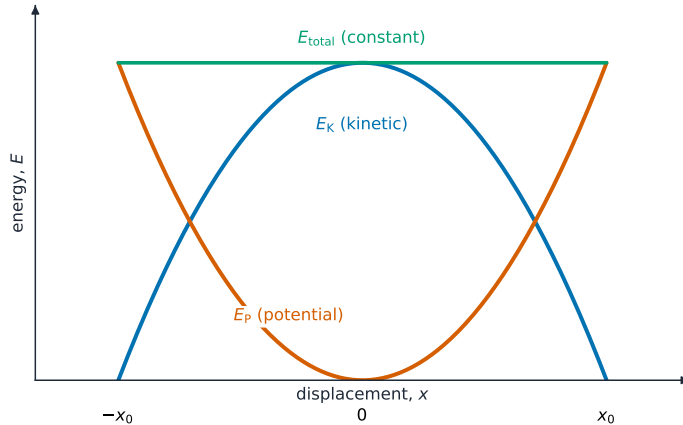
Set by the amplitude

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

All **potential** 弹性势能 at the extremes; all **kinetic** 动能 in the middle – solve for the speed at any x .



Energy of Simple Harmonic Oscillators



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – where is the energy split evenly?

At what displacement is the potential energy exactly half the total?

- $\frac{1}{2}kx^2 = \frac{1}{2}E = \frac{1}{4}kA^2$
- $x^2 = \frac{1}{2}A^2$
- $x = A/\sqrt{2} \approx 0.71A$

Much closer to the **end** than to the middle – because U grows as x^2 , so the outer part of the swing holds most of the potential energy.

5

Pendulums

Simple and physical pendulums

Simple pendulum (small angle)

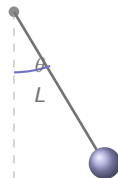
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

d = pivot to centre of mass.



1 m pendulum: $T \approx 2.0$ s.

The pendulum, and why small angles matter

small-angle approximation

小角度近似

$\sin \theta \approx \theta$ in radians – accurate to 1% below about 14°

the equation

$\tau = I\alpha$ gives $-mgL \sin \theta = I\ddot{\theta}$, which is SHM **only** after that approximation

simple pendulum

$T = 2\pi\sqrt{L/g}$ – mass cancels, amplitude does not appear

physical pendulum

$T = 2\pi\sqrt{\frac{I}{mgd}}$, with d from pivot to centre of mass

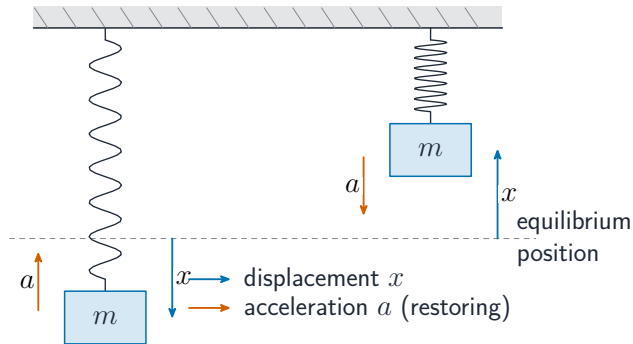
natural

frequency 固有频率

$f_0 = \frac{1}{2\pi}\sqrt{k/m}$; driving at f_0 gives **resonance** 共振 and a growing amplitude

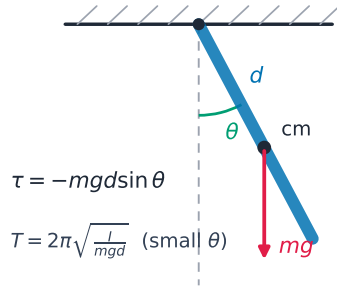
At large angles a pendulum is **not** simple harmonic and its period grows with amplitude. Every quoted pendulum formula silently assumes the approximation.

Defining Simple Harmonic Motion



Simple harmonic motion 简谐运动 needs exactly two things: an **equilibrium** 平衡位置 point, and a restoring force **proportional to the displacement** and directed back towards it. Recognise the equation $\ddot{x} = -\omega^2 x$ and you know the motion is sinusoidal – you are not asked to solve it.

The restoring torque on a physical pendulum



physical pendulum: $T = 2\pi \sqrt{I/mgd}$

Gravity acting at the center of mass provides a physical pendulum's restoring torque

The phase constant, and one more oscillator

Phase constant

In $x = A \cos(\omega t + \phi)$ the **phase constant** ϕ sets where the motion starts: $\phi = 0$ from the amplitude, $\phi = -\pi/2$ from the equilibrium moving positive.

Reading it off

Two initial conditions – $x(0)$ and $v(0)$ – fix A and ϕ together.

Torsion pendulum

A **torsion pendulum** twists a wire instead of stretching a spring: $\tau = -\kappa\theta$, so $\omega = \sqrt{\kappa/I}$.

The pattern

Every simple harmonic oscillator has the same form: a restoring quantity over an inertial one, square-rooted.

Identify the restoring constant and the inertia, and ω follows – k/m , κ/I , mgd/I for a physical pendulum.

Exam tips

- Identify SHM from a **linear restoring force** $F = -kx$, which gives $\frac{d^2x}{dt^2} = -\omega^2x$ with $\omega = \sqrt{k/m}$.
- Write the solution $x = A \cos(\omega t + \phi)$ and get v, a by differentiating; period $T = \frac{2\pi}{\omega}$ is amplitude-independent.
- Energy trades between $\frac{1}{2}kx^2$ and $\frac{1}{2}mv^2$, with total $\frac{1}{2}kA^2$.
- For a pendulum, use the small-angle approximation $\sin \theta \approx \theta$ to reach SHM.
- Match phase to the start: released from rest at maximum displacement uses cosine.

6

Recap

Unit 7 – key takeaways

1

SHM

$$d^2x/dt^2 = -\omega^2x \text{ with } \omega = \sqrt{k/m}$$

2

Motion

$$x = A \cos(\omega t + \phi); v_{\max} = A\omega, \\ a_{\max} = A\omega^2$$

3

Energy

$$E = \frac{1}{2}kA^2; \text{ trades between PE} \\ \text{and KE}$$

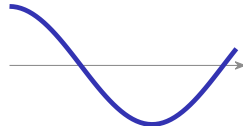
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Pendulums

$$2\pi\sqrt{L/g} \text{ (simple), } 2\pi\sqrt{I/mgd} \\ \text{(physical)}$$

Check yourself

- 1 What differential equation defines simple harmonic motion?
- 2 Does a mass-spring period depend on the amplitude?
- 3 Where in the swing is the speed greatest?
- 4 Does a simple pendulum's period depend on the bob's mass?



Answers 1. $d^2x/dt^2 = -\omega^2x$ 2. no 3. in the middle 4. no