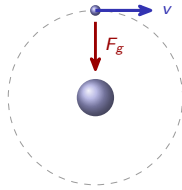


Energy & Momentum of Rotating Systems

AP Physics C: Mechanics ■ Unit 6

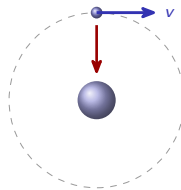
AP Physics C: Mechanics



Energy & Momentum of Rotating Systems

What we will cover

- 1 Rotational kinetic energy
- 2 Work and power of a torque
- 3 Angular momentum and impulse
- 4 Conservation of angular momentum
- 5 Rolling without slipping
- 6 Orbiting satellites



1 Rotational KE

Rotational kinetic energy

Energy of spin

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

A rolling body has **both**: $K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$.

A **flywheel** 飞轮 stores energy in its spin – and $K \propto \omega^2$.

Torque and work

$$W = \int \tau d\theta, \quad P = \tau \omega$$

Area under a τ - θ graph; $W_{\text{net}} = \Delta K_{\text{rot}}$.

$\tau = 15$, $\Delta\theta = 8$: $W = 120$ J; at $\omega = 20$,
 $P = 300$ W.

2 Angular momentum

Angular momentum and impulse

Angular momentum 角动量 is the rotational twin of momentum:

Point particle and rigid body

$$\vec{L} = \vec{r} \times \vec{p}, \quad L = I \omega$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

Angular impulse 角冲量 $\int \tau dt = \Delta L$.

Worked example

$L_i = 30$; $\tau = 5$ N m for 4 s:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50 \text{ kg m}^2/\text{s}$$

Conservation of angular momentum

With **zero net external torque**, angular momentum is conserved – even as the shape changes:

Before = after

$$I_i \omega_i = I_f \omega_f$$

A skater pulls their arms in: I drops, so ω rises. (Kinetic energy rises too – paid by muscular work.)



Skater spin

3 Rolling

Rolling without slipping

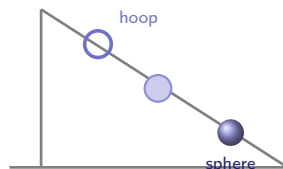
Rolling without slipping 无滑滚动 links translation and rotation:

The condition

$$v_{cm} = \omega r, \quad a_{cm} = \alpha r$$

Static friction 静摩擦 produces the rolling but does **no work** (the contact point is momentarily at rest).

Down a ramp: the solid sphere wins, the hoop last.



4 Satellites

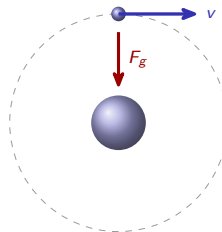
Orbiting satellites

Gravity supplies the centripetal force. Set them equal:

Orbital speed and Kepler

$$v = \sqrt{\frac{GM}{r}}, \quad T^2 \propto r^3$$

- energy $E = K + U$, with $U = -GMm/r$
- elliptical orbits conserve energy *and* angular momentum



A bigger orbit means
a **slower** speed, longer period.

5 Recap

Unit 6 – key takeaways

1 Rotational energy

$$K_{\text{rot}} = \frac{1}{2} I \omega^2; W = \int \tau d\theta$$

2 Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = I\omega; \vec{\tau} = d\vec{L}/dt$$

3 Conservation

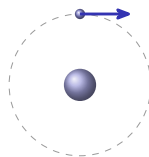
$$\text{no external torque} \Rightarrow I_i \omega_i = I_f \omega_f$$

4 Orbits

$$v = \sqrt{GM/r}; \text{Kepler's } T^2 \propto r^3$$

Check yourself

- 1 Write the net torque as a rate of change of angular momentum.
- 2 A skater pulls their arms in – what happens to ω ?
- 3 Does the static friction on a rolling wheel do work?
- 4 A satellite moves to a bigger orbit – faster or slower?



Answers 1. $\vec{\tau} = d\vec{L}/dt$ 2. it speeds up 3. no 4. slower