Energy & Momentum of Rotating Systems

AP Physics C: Mechanics ▪ Unit 6

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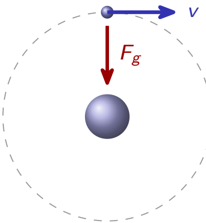
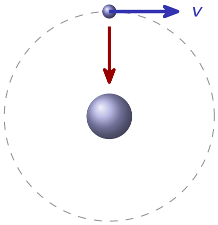


What we will cover

1. Rotational kinetic energy
2. Work and power of a torque
3. Angular momentum and impulse
4. Conservation of angular momentum
5. Rolling without slipping
6. Orbiting satellites



Rotational kinetic energy

Energy of spin

$$K_{rot} = \frac{1}{2} I\omega^2$$

A rolling body has both: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

A **flywheel** 飞轮 stores energy in its spin – and $K \propto \omega^2$.

Torque and work

$$W = \int \tau \, d\theta, \qquad P = \tau\omega$$

Area under a τ–θ graph; $W_{net} = \Delta K_{rot}$.

$\tau = 15$, $\Delta\theta = 8$: $W = 120\,\mathrm{J}$; at $\omega = 20$, $P = 300\,\mathrm{W}$.

Angular momentum and impulse

Angular momentum 角动量 is the rotational twin of momentum:

Point particle and rigid body

$$\vec{L} = \vec{r} \times \vec{p}, \qquad L = I\omega$$

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Angular impulse 角冲量 $\int \tau \, dt = \Delta L$.

Worked example

$L_i = 30$; $\tau = 5\,\mathrm{N\,m}$ for $4\,\mathrm{s}$:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50\,\mathrm{kg\,m^2/s}$$

Conservation of angular momentum

With zero net external torque, angular momentum is conserved – even as the shape changes:

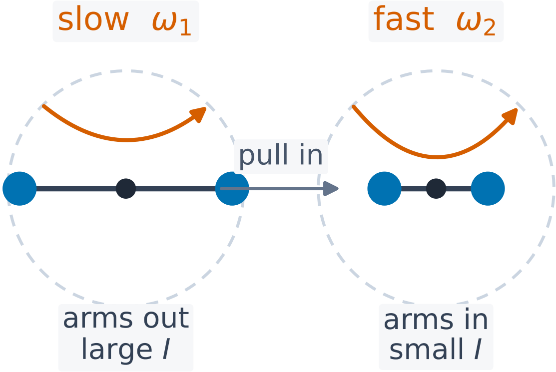
Before = after

$$I_i \omega_i = I_f \omega_f$$

A skater pulls their arms in: I drops, so ω rises. (Kinetic energy rises too – paid by muscular work.)

skater Spin

Conservation of angular momentum



$L = I\omega$ conserved (no external torque): $I_1\omega_1 = I_2\omega_2$

Rolling without slipping

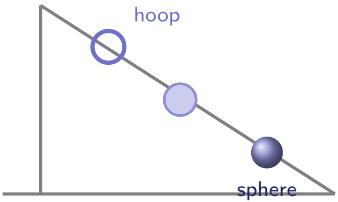
Rolling without slipping 无滑滚动 links translation and rotation:

The condition

$$v_{cm} = \omega r, \qquad a_{cm} = \alpha r$$

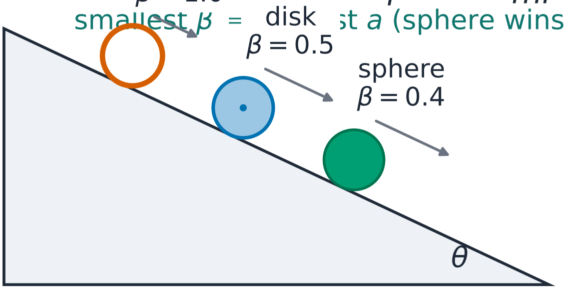
Static friction 静摩擦 produces the rolling but does no work (the contact point is momentarily at rest).

> Down a ramp: the solid sphere wins, the hoop last.



Rolling without slipping

$$a_{\mathrm{cm}} = \frac{g\sin\theta}{1+\beta}, \quad \beta = \frac{I}{mr^2}$$



rolling race: smaller I reaches bottom first

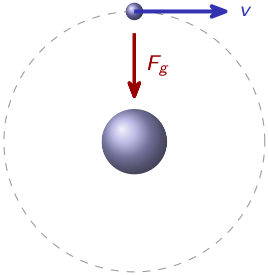
Orbiting satellites

Gravity supplies the centripetal force. Set them equal:

Orbital speed and Kepler

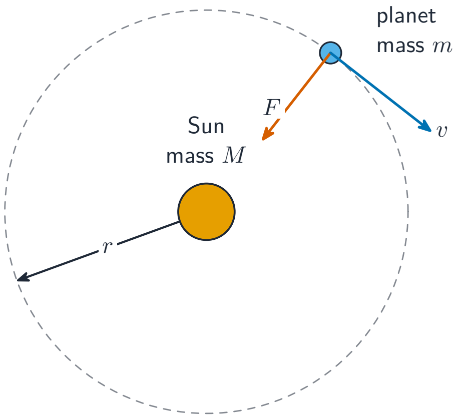
$$v = \sqrt{\frac{GM}{r}}, \qquad T^2 \propto r^3$$

- energy $E = K + U$, with $U = -GMm/r$
- elliptical orbits conserve energy *and* angular momentum



A bigger orbit means a slower speed, longer period.

Orbiting satellites



Unit 6 – key takeaways

1 Rotational energy

$$K_{rot} = \frac{1}{2} I\omega^2;\ W = \int \tau \, d\theta$$

2 Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = I\omega;\ \vec{\tau} = d\vec{L}/dt$$

3 Conservation

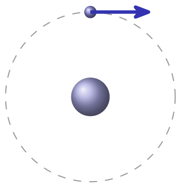
no external torque $\Rightarrow I_i\omega_i = I_f\omega_f$

4 Orbits

$v = \sqrt{GM/r}$; Kepler's $T^2 \propto r^3$

Check yourself

1. Write the net torque as a rate of change of angular momentum.
2. A skater pulls their arms in – what happens to ω?
3. Does the static friction on a rolling wheel do work?
4. A satellite moves to a bigger orbit – faster or slower?



Answers **1.** $\vec{\tau} = d\vec{L}/dt$ **2.** it speeds up **3.** no **4.** slower