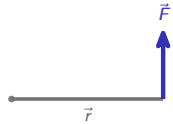


Torque & Rotational Dynamics

AP Physics C: Mechanics ■ Unit 5

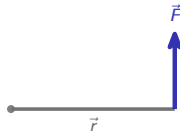
AP Physics C: Mechanics



Torque & Rotational Dynamics

What we will cover

- 1 Rotational kinematics
- 2 Linking linear and rotational motion
- 3 Torque as a cross product
- 4 Rotational inertia
- 5 Rotational equilibrium
- 6 Newton's second law for rotation



1

Rotational kinematics

Rotational kinematics

Angular motion mirrors the linear case – through calculus:

Derivatives

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

Constant- α equations have the same form as SUVAT; for a **variable** α , integrate: $\omega = \int \alpha dt$.

Worked example ($\alpha = 4t$)

$$\omega = \int_0^t 4t' dt' = 2t^2$$

$$\theta = \int_0^t 2t'^2 dt' = \frac{2}{3}t^3$$

Only calculus handles a non-constant α .

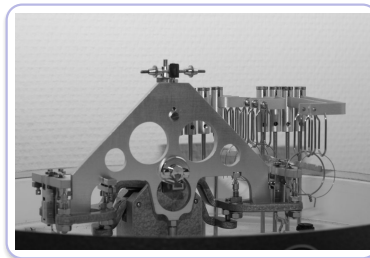
Linking linear and rotational motion

Every point on a **rigid body** 剛体 moves along a curve, tied to the radius r :

Same ω , different v

$$v = \omega r, \quad a_t = \alpha r$$

All points share one ω ; points farther out move **faster**.



Balance scale

2 Torque

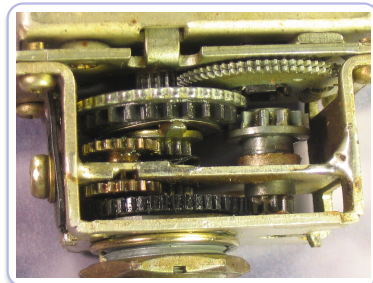
Torque

Torque 力矩 is the rotational effect of a force:

A cross product

$$\tau = rF \sin \theta, \quad \vec{\tau} = \vec{r} \times \vec{F}$$

The **lever arm** 力臂 is the perpendicular distance to the force – a force farther out and $\perp$ to $\vec{r}$ twists most.



Gear train

3 Rotational inertia

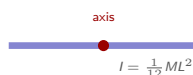
Rotational inertia

Rotational inertia 转动惯量 resists angular acceleration, and depends on **how mass is spread**:

Sum, or integrate

$$I = \sum m_i r_i^2 = \int r^2 dm$$

Parallel-axis theorem 平行轴定理: $I = I_{cm} + Md^2$.



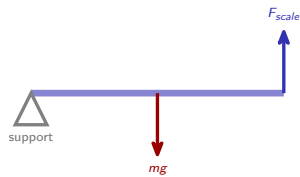
A rod is harder to spin about its end.

4 Equilibrium

Rotational equilibrium

In **rotational equilibrium** 转动平衡 the **net torque** 净力矩 is zero. Full **static equilibrium** 静平衡: both net force and net torque zero.

- draw an **extended free-body diagram** 扩展受力图 (where each force acts)
- put the axis at an unknown force to drop it out



$$mg \times 2 = F_{scale} \times 4 \Rightarrow F_{scale} = 49 \text{ N.}$$

5 Rotational Newton

Newton's second law for rotation

A nonzero net torque gives an angular acceleration – exactly like $F = ma$:

Rotational form

$$\tau_{\text{net}} = I\alpha$$

torque $\leftrightarrow$ force, $I \leftrightarrow$ mass, $\alpha \leftrightarrow$ acceleration.

Worked example

$\tau = 6 \text{ N m}$ on $I = 2 \text{ kg m}^2$:

$$\alpha = \frac{6}{2} = 3 \text{ rad/s}^2$$

Double $I \Rightarrow$ half the α . For a pulley, pair it with $F = ma$ (linked by $a = \alpha r$).

6

Recap

Unit 5 – key takeaways

1 Kinematics

$\omega = d\theta/dt$, $\alpha = d\omega/dt$; integrate for variable α

2 Torque

$\vec{\tau} = \vec{r} \times \vec{F}$; the lever arm sets the twist

3 Inertia

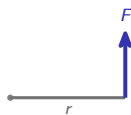
$I = \int r^2 dm$; mass far from the axis counts most

4 Dynamics

equilibrium: net $\tau = 0$; otherwise $\tau = I\alpha$

Check yourself

- 1 What integral gives the rotational inertia of a continuous body?
- 2 Is a rod harder to spin about its end or its center?
- 3 Torque is the cross product of which two vectors?
- 4 $\tau = 10 \text{ N m}$ on $I = 5 \text{ kg m}^2$ – find α .



Answers 1. $\int r^2 dm$ 2. its end 3. $\vec{r}$ and $\vec{F}$ 4. 2 rad/s^2