Torque & Rotational Dynamics

AP Physics C: Mechanics ▪ Unit 5

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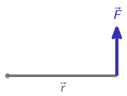
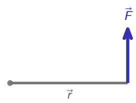


What we will cover



1 Rotational kinematics

Rotational kinematics

Angular motion mirrors the linear case – through calculus:

Derivatives

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt}$$

Constant-α equations have the same form as SUVAT; for a variable α, integrate: $\omega = \int \alpha \, dt$.

Worked example ($\alpha = 4t$)

$$\omega = \int_0^t 4t' \, dt' = 2t^2$$

$$\theta = \int_0^t 2t'^2 \, dt' = \tfrac{2}{3}t^3$$

Only calculus handles a non-constant α.

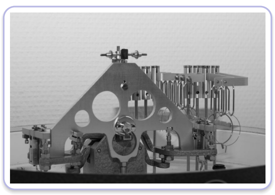
Linking linear and rotational motion

Every point on a **rigid body** 刚体 moves along a curve, tied to the radius r:

Same ω, different v

$$v = \omega r, \qquad a_t = \alpha r$$

All points share one ω; points farther out move faster.



Balance scale

2 Torque

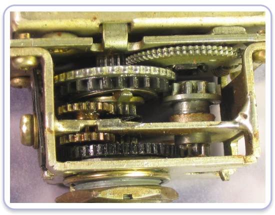
Torque

Torque 力矩 is the rotational effect of a force:

A cross product

$$\tau = rF\sin\theta, \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

The **lever arm** 力臂 is the perpendicular distance to the force – a force farther out and $\perp$ to $\vec{r}$ twists most.



Gear train

3 Rotational inertia

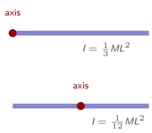
Rotational inertia

Rotational inertia 转动惯量 resists angular acceleration, and depends on how mass is spread:

Sum, or integrate

$$I = \sum m_i r_i^2 = \int r^2\, dm$$

Parallel-axis theorem 平行轴定理: $I = I_{cm} + Md^2$.



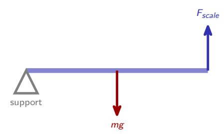
A rod is harder to spin about its end.

4 Equilibrium

Rotational equilibrium

In **rotational equilibrium** 转动平衡 the **net torque** 净力矩 is zero. Full **static equilibrium** 静平衡: both net force and net torque zero.

- draw an **extended free-body diagram** 扩展受力图 (where each force acts)
- put the axis at an unknown force to drop it out



$mg \times 2 = F_{scale} \times 4 \Rightarrow F_{scale} = 49\,\mathrm{N}.$

5 Rotational Newton

Newton's second law for rotation

A nonzero net torque gives an angular acceleration – exactly like $F = ma$:

Rotational form

$$\tau_{net} = I\alpha$$

torque ↔ force, I ↔ mass, α ↔ acceleration.

Worked example

$\tau = 6\,\mathrm{N\,m}$ on $I = 2\,\mathrm{kg\,m^2}$:

$$\alpha = \frac{6}{2} = 3\,\mathrm{rad/s^2}$$

Double $I \Rightarrow$ half the α. For a pulley, pair it with $F = ma$ (linked by $a = \alpha r$).

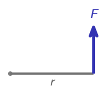
6 Recap

Unit 5 – key takeaways

1. **Kinematics**
   $\omega = d\theta/dt$, $\alpha = d\omega/dt$; integrate for variable α
2. **Torque**
   $\vec{\tau} = \vec{r} \times \vec{F}$; the lever arm sets the twist
3. **Inertia**
   $I = \int r^2\,dm$; mass far from the axis counts most
4. **Dynamics**
   equilibrium: net $\tau = 0$; otherwise $\tau = I\alpha$

Check yourself

1. What integral gives the rotational inertia of a continuous body?
2. Is a rod harder to spin about its end or its center?
3. Torque is the cross product of which two vectors?
4. $\tau = 10\,\mathrm{N\,m}$ on $I = 5\,\mathrm{kg\,m^2}$ – find α.



Answers **1.** $\int r^2\,dm$ **2.** its end **3.** $\vec{r}$ and $\vec{F}$ **4.** $2\,\mathrm{rad/s^2}$