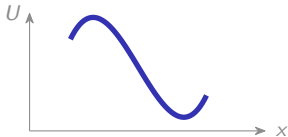


Work, Energy, and Power

AP Physics C: Mechanics ■ Unit 3

AP Physics C: Mechanics



Energy methods: from evidence to equations

Choose an object or system, identify energy transfers and write the relevant principle before calculating.

Practise signed work, variable-force integration, potential-energy graphs, two-spring energy accounting and power. Distinguish net work from one force's work, and a graph's area from its slope.

1

Translational Kinetic Energy

Kinetic energy and reference frame

$$K = \frac{1}{2}mv^2$$

Kinetic energy 动能 is a **scalar** 标量, measured in **joules** 焦耳. It depends on speed squared, not the sign of velocity.

A passenger at rest in a train moving at 10 m/s has zero translational kinetic energy in the train's **reference frame** 参考系, but nonzero energy in the platform frame. Use one frame consistently.

Speed and direction: original attempt

A 2.0 kg particle moves at $+3.0$ m/s, then at $+6.0$ m/s. Calculate its kinetic energy at each speed.

What would change if its velocity were -3.0 m/s instead?

Explain why a force can change direction without changing kinetic energy.

Kinetic energy: worked comparison

$$K = \frac{1}{2}mv^2$$

$$K_1 = \frac{1}{2}(2.0 \text{ kg})(3.0 \text{ m/s})^2 = 9.0 \text{ J}$$

$$K_2 = \frac{1}{2}(2.0 \text{ kg})(6.0 \text{ m/s})^2 = 36 \text{ J}$$

Doubling speed quadruples energy. At -3.0 m/s the squared speed still gives 9.0 J . A perpendicular force can change direction while doing zero work.

2

Work

Work: component and displacement

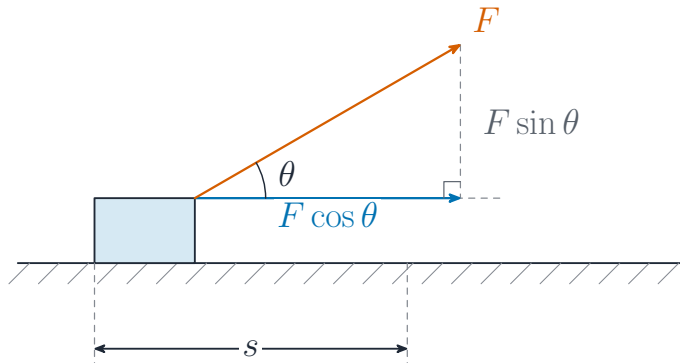
$$W = \int_a^b \vec{F} \cdot d\vec{r}$$

$$W = Fd \cos \theta \quad \text{for a constant force}$$

Work 功 transfers energy by a force through displacement of its point of application. The angle is between force and displacement.

The **integral** 积分 follows the actual path. In one dimension, signed area under $F_x(x)$ gives work; area below the axis contributes negatively.

Resolve the force along the displacement



For the pictured horizontal displacement, the parallel component contributes work; the perpendicular component does not.

Work signs: classify the geometry

Force and motion	Work and condition
Along displacement	Positive work by that force; other forces may still give negative net work.
Opposes displacement	Negative work by that force.
Perpendicular	Zero work; a normal force has this result only when the relevant displacement is perpendicular.
Friction	Negative on a block sliding over a fixed rough surface, but can be positive on an object accelerated by a moving belt.

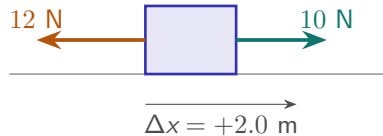
friction 摩擦力 and **normal force** 法向力 do not have fixed work signs by name.

Positive pull, falling speed: original attempt

A particle moves 2.0 m rightward. A constant 10 N pull acts rightward, while a constant 12 N resistance acts leftward.

Calculate work by each force and the net work.
Does the positive pulling work establish that the particle speeds up?

Assume enough initial kinetic energy to cover the displacement.



Signed work: worked check

$$W = Fd \cos \theta$$

$$W_{\text{pull}} = (10 \text{ N})(2.0 \text{ m}) \cos 0 = 20 \text{ J}$$

$$W_{\text{res}} = (12 \text{ N})(2.0 \text{ m}) \cos 180^\circ = -24 \text{ J}$$

$$W_{\text{net}} = W_{\text{pull}} + W_{\text{res}} = 20 \text{ J} - 24 \text{ J} = -4 \text{ J}$$

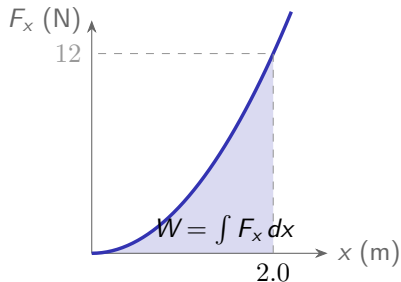
The **work–energy theorem** 动能定理 gives $\Delta K = W_{\text{net}}$. The particle loses kinetic energy despite positive work by the pull.

Variable force: original attempt

Along the x direction, $F_x = ax^2$ with $a = 3.0$ N/m². The particle travels from $x = 0$ to $L = 2.0$ m.

Integrate to find work. Explain why the coefficient's units matter.

If this is the net force on a 1.0 kg particle initially at rest, find its final speed.



Variable-force work: symbolic derivation

$$W = \int_0^L F_x dx$$

$$W = \int_0^L ax^2 dx = \left[\frac{ax^3}{3} \right]_0^L = \frac{aL^3}{3}$$

$$W = \frac{aL^3}{3} = \frac{(3.0 \text{ N/m}^2)(2.0 \text{ m})^3}{3} = 8.0 \text{ J}$$

$(\text{N/m}^2)(\text{m}^3) = \text{N m} = \text{J}$. The area is not a rectangle with the final force across the whole displacement.

From net work to final speed

$$W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2W_{\text{net}}}{m}}$$

$$v_f = \sqrt{0 + \frac{2(8.0 \text{ J})}{1.0 \text{ kg}}} = 4.0 \text{ m/s}$$

This result uses the force as net force and release from rest. If other forces do work, include their contributions first.

3

Potential Energy

Potential belongs to an interaction

$$\Delta U = - \int_a^b \vec{F}_c \cdot d\vec{r}$$

Potential energy 势能 describes configuration. **Conservative forces** 保守力 have path-independent work and an associated potential; their work around a closed path is zero.

Choose the zero of U for convenience. Adding a constant changes neither the force nor energy differences. For multiple interacting objects, count each pair once.

Three useful potentials

Interaction	Relation and assumption
Near-surface gravity	$\Delta U_g = mg\Delta y$ with approximately constant g .
Spherical masses	$U_g = -Gm_1 m_2 / r$ with zero at infinite separation.
Ideal spring	$U_s = \frac{1}{2} kx^2$, with x measured from natural length.
System choice	Include Earth or the spring when accounting for the corresponding interaction potential.

Force from a potential-energy graph

$$F_x = -\frac{dU}{dx}$$

A **derivative** 导数 gives slope: positive slope means negative force; negative slope means positive force.

Zero slope gives zero force at that position, not necessarily zero speed. A local minimum gives **stable equilibrium** 稳定平衡; a local maximum gives **unstable equilibrium** 不稳定平衡 in the usual smooth one-dimensional cases.

Cubic potential: original attempt

$$U(x) = ax^3 - bx$$

$a = 2.0 \text{ J/m}^3$, $b = 6.0 \text{ J/m}$. Derive the force, find equilibrium positions and classify their stability.

Find the force at $x = 0$. Why is the particle's speed not determined by the slope alone?

Cubic potential: differentiate and solve

$$F_x = -\frac{dU}{dx} = b - 3ax^2$$

$$F_x = 0 \quad \Rightarrow \quad x_{\text{eq}} = \pm \sqrt{\frac{b}{3a}}$$

$$x_{\text{eq}} = \pm \sqrt{\frac{6.0 \text{ J/m}}{3(2.0 \text{ J/m}^3)}} = \pm 1.0 \text{ m}$$

These positions give zero force. The energy and initial conditions still determine whether the particle passes them with nonzero speed.

Cubic potential: stability and force direction

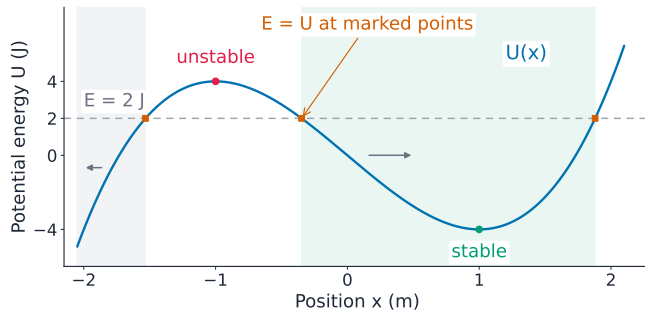
$$U''(x) = 6ax$$

At $x = +1.0$ m, curvature is positive: a minimum and stable equilibrium. At $x = -1.0$ m, curvature is negative: a maximum and unstable equilibrium.

$$F_x(0) = b - 3a(0)^2 = b = 6.0 \text{ J/m} = 6.0 \text{ N}$$

The force at zero points in the positive direction. Neither this force nor equilibrium classification fixes the speed.

Potential, force and separate allowed intervals



Shading: $U \leq E$. Separate allowed intervals need not connect.
 Force arrows follow $F = -dU/dx$; the curve uses $a = 2$ J/m³ and $b = 6$ J/m.

Read slope for force and the gap $E - U$ for kinetic energy. Allowed intervals can be disconnected.

4

Conservation of Energy

Choose the system before the energy balance

Mechanical energy 机械能 is $K + U$. In the course's particle model, one object has kinetic energy; include Earth or the spring for interaction potential. Real bodies can also have internal energy.

$$K_i + U_i = K_f + U_f$$

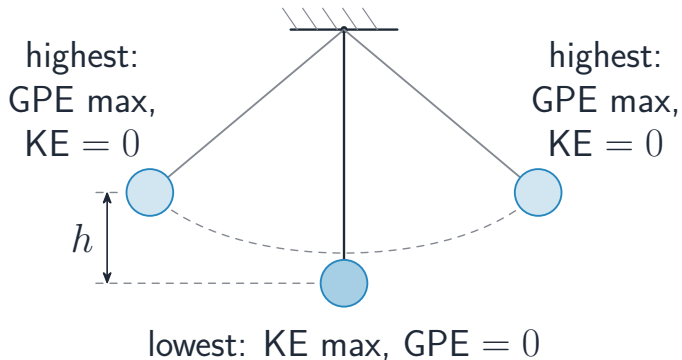
ideal isolated mechanical system, no dissipation

Nonconservative interactions can transfer or dissipate mechanical energy; they do not invariably remove it.

One fall, two consistent accounts

Chosen system	Energy account
Particle alone	Gravity is external: $W_g = \Delta K$ if it is the only force doing work.
Particle + Earth	Gravity is internal: $\Delta K + \Delta U_g = 0$ with negligible external transfer.
Avoid double counting	Do not include both gravitational work and the same gravitational potential change as separate transfers.
Boundary	External work equals system energy change only when work is the sole transfer mechanism.

Ideal pendulum energy exchange



For release from rest with negligible dissipation, extreme positions have zero speed and the lowest position has greatest kinetic energy.

Friction and thermal accounting

For a block sliding over a fixed rough track, constant kinetic friction dissipates mechanical energy:

$$\Delta E_{\text{mech}} = -f_k s$$

With block, Earth and track inside an isolated system, and only these changes relevant:

$$0 = \Delta K + \Delta U + \Delta E_{\text{thermal}}$$

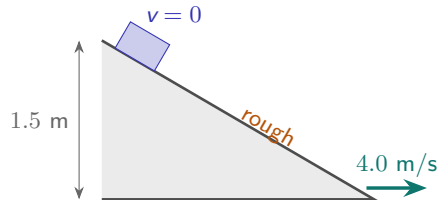
Nonconservative forces 非保守力 can produce **thermal energy** 热能. Variable friction needs an integral; other transfers need additional terms.

Rough ramp: original attempt

A 2.0 kg block starts from rest at height 1.5 m and reaches the bottom at 4.0 m/s. Take $g = 9.8 \text{ m/s}^2$.

Choose block, Earth and track as the system.
Assume no energy transfer to the surroundings
and no other changing energy store.

Calculate the thermal-energy increase. State
which assumption allows that interpretation.



Ramp energy: calculate the stores

$$U_i = mgh$$

$$U_i = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = 29.4 \text{ J}$$

$$K_f = \frac{1}{2}mv_f^2$$

$$K_f = \frac{1}{2}(2.0 \text{ kg})(4.0 \text{ m/s})^2 = 16.0 \text{ J}$$

Ramp balance: identify the destination

$$0 = \Delta K + \Delta U + \Delta E_{\text{thermal}}$$

$$\Delta E_{\text{thermal}} = U_i - K_f$$

$$\Delta E_{\text{thermal}} = 29.4 \text{ J} - 16.0 \text{ J} = 13.4 \text{ J}$$

The mechanical-energy decrease is thermal increase under this specified model. If sound or energy transfer out mattered, the same subtraction would not establish thermal increase alone.

Allowed positions and turning points

$$K = E - U(x) \geq 0$$

For fixed total mechanical energy in one dimension, $U > E$ is inaccessible. Ordinary **turning points** 转折点 occur at the limits of bounded motion where speed reaches zero and direction reverses.

Equality $E = U$ alone is insufficient: a particle at rest at a stable minimum with $E = U_{\min}$ stays there. Energy alone does not specify velocity direction or connect separated allowed intervals.

Energy gap: original graph task

For the cubic potential, take $E = 2.0$ J and particle mass 2.0 kg. At $x = 1.0$ m, the potential is -4.0 J.

Find kinetic energy and speed there. Identify why this is the fastest point within the bounded well.

Explain why the same statement need not hold across the disconnected allowed region farther left.

Energy gap: worked check

$$K = E - U$$

$$K = 2.0 \text{ J} - (-4.0 \text{ J}) = 6.0 \text{ J}$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.0 \text{ J})}{2.0 \text{ kg}}} \approx 2.45 \text{ m/s}$$

U is lowest here within the bounded well. This limits the claim to the connected motion region, not every allowed point of the cubic potential.

Roller coaster: evidence and idealisation



Roller coaster track

Height affects gravitational energy.

Speed also depends on initial energy, propulsion, braking and dissipation.

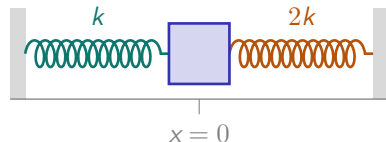
A photograph does not establish that the lowest pictured point is necessarily the fastest. Use the stated physical model.

2025 Q2: verified two-spring setup

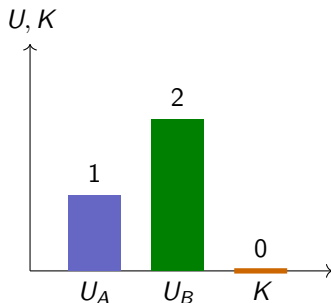
In Version J, a block lies between two springs attached to fixed walls. Their constants are k and $2k$; both are relaxed at $x = 0$. Each changes length by magnitude $|x|$.

The block is held at $x = x_1 > 0$ and released from rest, with negligible friction.

For part A, compare U_A , U_B and K . For part B, derive the speed at $x = x_1/2$, beginning with a fundamental principle as the actual prompt requests.



Two-spring energy bars: worked check



$$U_A = \frac{1}{2}kx_1^2$$

$$U_B = \frac{1}{2}(2k)x_1^2 = 2U_A$$

At release, $K = 0$. Use an explicit line at zero. The bar units are multiples of $\frac{1}{2}kx_1^2$; both elastic energies are positive.

Two-spring derivation: initial and final energies

$$K_i + U_{A,i} + U_{B,i} = K_f + U_{A,f} + U_{B,f}$$

$$E_i = \frac{1}{2} k x_1^2 + \frac{1}{2} (2k) x_1^2 = \frac{3}{2} k x_1^2$$

$$U_f = \frac{1}{2} k \left(\frac{x_1}{2} \right)^2 + \frac{1}{2} (2k) \left(\frac{x_1}{2} \right)^2 = \frac{3}{8} k x_1^2$$

Halving deformation quarters each spring's potential energy. Include both springs, and do not confuse compression with negative elastic potential.

Two-spring derivation: solve for speed

$$K_f = E_i - U_f = \frac{3}{2}kx_1^2 - \frac{3}{8}kx_1^2 = \frac{9}{8}kx_1^2$$

$$\frac{1}{2}mv^2 = \frac{9}{8}kx_1^2$$

$$v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$$

This agrees with the official 2025 solution for positive x_1 . A speed is nonnegative; for arbitrary signed release coordinates, use the magnitude. The actual prompt specifies starting from a principle.

5

Power

Average and instantaneous power

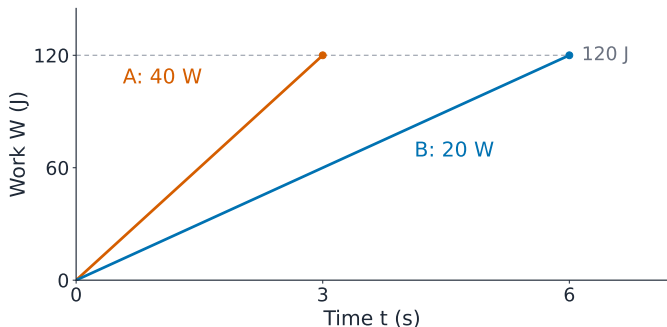
$$P_{\text{avg}} = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}$$

$$P_{\text{inst}} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Power 功率 is energy transfer or conversion per unit time, measured in **watts** 瓦特 (J/s).

On a work–time graph, a secant gives interval-average power and a tangent gives instantaneous power. They agree on a straight line, but not on every curve.

Power from work–time slopes



Original example: $P = \Delta W / \Delta t = dW/dt$ on each straight segment.
The same 120 J in less time requires greater constant power.

For these straight segments, each slope is constant: average and instantaneous power agree within the segment.

Steady lifting: original attempt

Lift a 50 kg load steadily upward at 2.0 m/s, with $g = 9.8 \text{ m/s}^2$ and negligible losses.

Find the lifting force and power. Predict the power at twice the speed.

Does lifting to the same height then require twice the work? Explain the difference between energy and rate.

Steady lifting: worked check

$$F = mg = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490 \text{ N}$$

$$P = Fv = (490 \text{ N})(2.0 \text{ m/s}) = 980 \text{ W}$$

At 4.0 m/s, ideal power doubles to 1960 W. For the same final height, ideal work remains mgh ; the transfer occurs in half the time.

Steady speed and negligible losses are assumptions, not consequences of the word “lift”.

Car climbing a hill: original attempt

A 1200 kg car travels steadily at 15 m/s along a road rising 1.0 m for each 20 m of road distance.

Using $g = 9.8 \text{ m/s}^2$, calculate vertical speed, then power against gravity.

What additional information would be needed to find engine input power including losses?

Hill power: geometry and energy rate

$$v_y = v \frac{\Delta y}{\Delta s}$$

$$v_y = (15 \text{ m/s}) \frac{1.0 \text{ m}}{20 \text{ m}} = 0.75 \text{ m/s}$$

$$P = mgv_y$$

$$P = (1200 \text{ kg})(9.8 \text{ m/s}^2)(0.75 \text{ m/s}) = 8820 \text{ W}$$

About 8.8 kW against gravity. Drag, rolling losses and efficiency would affect required input power.

Ramp to motor: original transfer task

A particle starts from rest and descends through 3.0 m on a fixed frictionless ramp. Assume it stays on the track and has no other energy transfer.

Find its bottom speed. A motor then maintains that speed against a 20 N resistance: find ideal power.

State what changes if friction or loss of contact invalidates the assumed motion.

Ramp to motor: worked check

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

$$v = \sqrt{2(9.8 \text{ m/s}^2)(3.0 \text{ m})} \approx 7.7 \text{ m/s}$$

$$P = Fv = (20 \text{ N})(7.7 \text{ m/s}) \approx 150 \text{ W}$$

The speed uses the height drop and ideal energy balance, not ramp length. The motor result uses steady motion and the specified resistance only.

Varying power: original attempt

$$W(t) = ct^2, \quad c = 2.0 \text{ J/s}^2$$

From $t = 0$ to 3.0 s, calculate average power. Find instantaneous power at 3.0 s.

Explain why using the final tangent slope as the whole interval's average would be wrong.

Average versus instantaneous: worked check

$$P_{\text{avg}} = \frac{W(t_f) - W(0)}{t_f} = ct_f$$

$$P_{\text{avg}} = (2.0 \text{ J/s}^2)(3.0 \text{ s}) = 6.0 \text{ W}$$

$$P_{\text{inst}} = \frac{dW}{dt} = 2ct$$

$$P_{\text{inst}}(3.0 \text{ s}) = 2(2.0 \text{ J/s}^2)(3.0 \text{ s}) = 12 \text{ W}$$

The final rate exceeds the interval average because the rate increases with time.

6

Exam tips

Energy: retrieval

- 1 Can a pull do positive work while kinetic energy falls?
- 2 Does zero slope on $U(x)$ require zero speed?
- 3 Why must both spring energies appear in the 2025 derivation?
- 4 How do area, slope and energy gap answer different graph questions?

Energy: retrieval checks

Question	Reasoning
Work signs	Net work, not one contribution, determines the kinetic-energy change.
Equilibrium	Zero slope gives zero force; speed also depends on energy and initial conditions.
Two springs	Both deformations store positive elastic energy; halving each deformation quarters each store.
Graphs	Area of $F_x(x)$ gives work; slope of $U(x)$ gives $-F_x$; $E - U$ gives K ; slope of $W(t)$ gives power.

Check yourself

Original transfer task

A motor pulls a block uphill on a rough fixed track. The block moves at steady speed and becomes warmer.

Choose a system and write an energy balance with clearly stated transfers and stores. Explain how positive motor work can coexist with zero change in kinetic energy. State what additional measurements would let you find the motor's power.