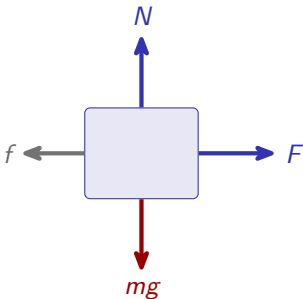


Force & Translational Dynamics

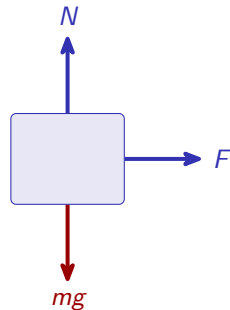
AP Physics C: Mechanics ■ Unit 2

AP Physics C: Mechanics



What we will cover

- 1 Systems and center of mass
- 2 Free-body diagrams and Newton's laws
- 3 Gravitation
- 4 Friction
- 5 Spring forces
- 6 Resistive forces and terminal velocity
- 7 Circular motion



1

Center of mass

Systems and center of mass

A **system** 系统 moves as if all its mass sat at its **center of mass** 质心:

Discrete and continuous

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1}{M} \int x dm$$

Only the **net external force** 净外力 accelerates the center of mass – internal forces cannot.

Worked example – a uniform rod

With $\lambda = M/L$, $dm = \lambda dx$:

$$x_{cm} = \frac{1}{M} \int_0^L x \lambda dx = \frac{L}{2}$$

The integral confirms it balances at the middle.

2

Newton's laws

Free-body diagrams and Newton's laws

Draw every force on a **free-body diagram** 受力图, resolve into components, add to the **net force** 合力.

The three laws

1st: inertia $\vec{F}_{net} = m\vec{a}$

3rd: equal-opposite pairs, different bodies



Thrust up, weight down: the net force sets the acceleration

Worked example – two bodies, one string

A 6.0 kg cart on a frictionless table is pulled by a light string over a pulley to a hanging 2.0 kg mass. Find a and the tension.

- treat them as one system:

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(2.0)(9.8)}{8.0} = 2.45 \text{ m/s}^2$$

- then one body alone for T : cart,

$$T = m_1 a = 14.7 \text{ N}$$

- check with the hanging mass:

$$m_2 g - T = 19.6 - 14.7 = 4.9 = m_2 a \checkmark$$

The string constraint – same $|a|$, same $|T|$ – is what lets you do this. Solve for a with the whole system, then use one body to get an internal force.

Newton's laws, one axis at a time

third law 牛顿第三定律

A pushes B, B pushes back equally and oppositely. The pair acts on **different objects**, so it never cancels inside one free-body diagram

second law

$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$ for constant mass – apply it **one axis at a time**

when F depends on v

$m dv/dt = F(v)$ is a differential equation; separate and integrate

translational equilibrium 平动平衡

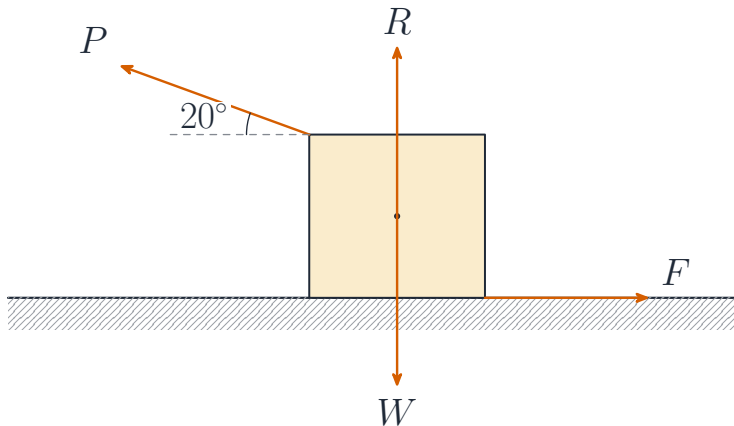
$\sum \vec{F} = 0$, so $\vec{a} = 0$ – at rest *or* at constant velocity

normal force 法向力

perpendicular to the surface; it is whatever it needs to be, and is not always mg

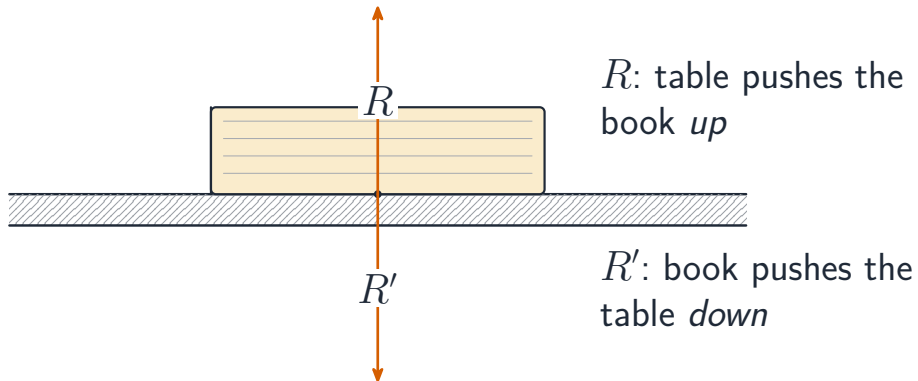
An action–reaction pair on one diagram is the commonest error in the unit. If both arrows are on the same body, one of them does not belong there.

Forces and Free-Body Diagrams



A free-body diagram shows every force acting on one object

A Newton's third-law pair



A Newton's third-law pair: equal and opposite forces on two different objects

3

Gravity

Gravitation

Universal gravitation

$$F_g = \frac{Gm_1m_2}{r^2}, \quad g = \frac{GM}{r^2}$$

- **mass** 质量 is intrinsic; **weight** 重量 $= mg$
- a sphere pulls as if all mass sat at its centre
- g falls off as $1/r^2$ with **altitude** 海拔

Worked example

At one Earth radius up, r doubles:

$$g = \frac{9.8}{2^2} = 2.45 \text{ m/s}^2$$

The gentler pull is what lets satellites orbit high up.

4

Friction & springs

Friction and spring forces

Friction

$$f_s \leq \mu_s N, \quad f_k = \mu_k N$$

Depends on N and μ – not on the contact area.

Hooke's law

$$F_s = -kx$$

Combining springs: **parallel** adds ($k_{eq} = k_1 + k_2$, stiffer); **series** is softer ($\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$).

Two 100 N/m springs: parallel 200, series 50 N/m.

Worked example – a block sliding down a rough incline

A block slides down a 30° incline with $\mu_k = 0.20$. Find its acceleration.

- perpendicular: $N = mg \cos 30^\circ$
- along: $ma = mg \sin 30^\circ - \mu_k mg \cos 30^\circ$
- $a = g(\sin 30^\circ - 0.20 \cos 30^\circ) = 3.2 \text{ m/s}^2$

The mass cancels – the answer is the same for a marble and a boulder. Note $N \neq mg$ on an incline; using mg there is the standard lost mark.

Static and kinetic friction are not the same law

kinetic 动摩擦

$f_k = \mu_k N$ – a **fixed** value while sliding, opposing the slide

static 静摩擦

$f_s \leq \mu_s N$ – an **inequality**: it takes whatever value stops the slide, up to that maximum

usually

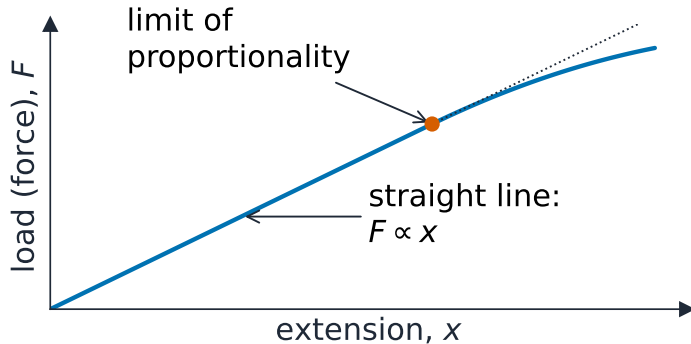
$\mu_s > \mu_k$, which is why a block is harder to start than to keep going

on an incline 斜面

resolve along and across the ramp: $N = mg \cos \theta$, and the driving component is $mg \sin \theta$

Writing $f_s = \mu_s N$ when nothing is on the point of slipping is wrong: static friction is only at its maximum at the instant of breaking away.

Hooke's law



$$F = kx \text{ (straight section)}$$
$$k = \text{gradient}$$

Hooke's law: extension is proportional to load up to the limit of proportionality

Combining springs, and the sign that trips everyone

in parallel

side by side, sharing the load: $k_{\text{eq}} = k_1 + k_2$ – stiffer than either

in series

end to end, sharing the stretch: $\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$ – softer than either

the spring
constant 彈簧常数

$F = -kx$; the minus sign says the force is a restoring one, always back toward natural length

the memory hook

exactly the *opposite* of resistors – springs in parallel add, resistors in series add

Two identical springs side by side hold twice the load at the same stretch; end to end they stretch twice as far under the same load.

5

Resistive forces

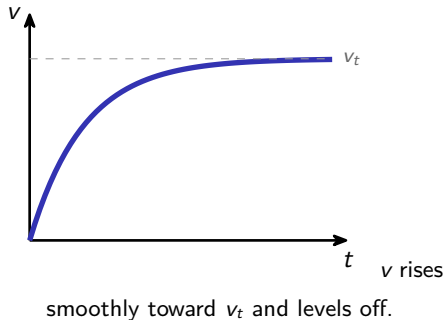
Resistive forces and terminal velocity

Drag grows with speed ($F = -bv$), so Newton's law becomes a **differential equation** 微分方程:

Falling with drag

$$m \frac{dv}{dt} = mg - bv$$

At **terminal velocity** 终极速度, $\frac{dv}{dt} = 0$, so $v_t = \frac{mg}{b}$.



Separating the variables, and why you must

the problem

with drag, $m \frac{dv}{dt} = mg - bv$ – the acceleration depends on v , so no SUVAT equation applies

separate the variables 分离变量

$$\int_0^v \frac{dv'}{mg - bv'} = \int_0^t \frac{dt'}{m}$$

the result

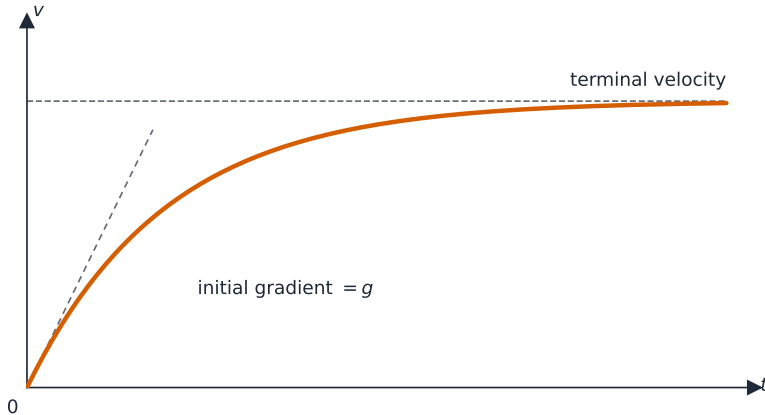
$$v(t) = \frac{mg}{b} (1 - e^{-bt/m})$$

terminal velocity

set $dv/dt = 0$: $v_T = \frac{mg}{b}$ – reachable without solving the equation at all

If a question asks only for the terminal value, set the acceleration to zero and stop. Integrate only when it asks for $v(t)$.

Terminal velocity in a fluid



An object falling through a fluid speeds up to a terminal velocity

6

Circular motion

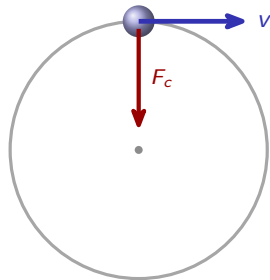
Circular motion

Uniform circular motion 匀速圆周运动:
steady speed, changing direction – so it
accelerates toward the centre:

Centripetal quantities

$$a_c = \frac{v^2}{r}, \quad F_c = \frac{mv^2}{r}$$

Non-uniform motion also has a **tangential** 切向
force that changes the speed.



A car needs F_c from
tyre friction – else it skids.

Circular motion needs a net inward force

**centripetal
acceleration** 向心加速度

$$a_c = \frac{v^2}{r} = \omega^2 r, \text{ always pointing to the centre}$$

the force

$F_c = \frac{mv^2}{r}$ – **not a new force**. It is the name for whatever the net inward force already is

identify it

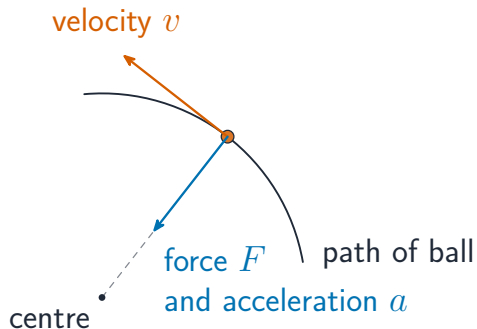
tension, gravity, friction, the normal force – say *which*, every time

**Newton's law of
gravitation**
万有引力定律

$$F = \frac{Gm_1m_2}{r^2} - \text{the inward force for an orbit}$$

There is no outward force. Writing 'centrifugal force' on a free-body diagram loses the mark; the sensation of being flung out is the body's **inertia**, not a force.

The velocity points along the tangent; the acceleration inward



The velocity points along the tangent; the centripetal force points to the centre

Gravitational Force

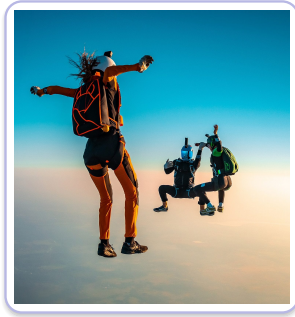

$$F = \frac{G m_1 m_2}{r^2}$$

$\vec{F}_{12} = -\vec{F}_{21}$ (third-law pair; equal magnitude, opposite direction)

$$F = Gm_1m_2/r^2 \quad \cdot \quad g = GM/r^2 \text{ outside sphere}$$

Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

Resistive Forces



Skydivers in freefall: drag grows with speed until it balances weight at terminal velocity

Exam tips

- Locate the **center of mass** with $x_{cm} = \frac{1}{M} \int x dm$ (or $\frac{\sum m_i x_i}{\sum m_i}$ for point masses) and use λ, σ, ρ for the mass element.
- The net external force moves the center of mass as if all mass sat there:
$$\vec{F}_{net} = M\vec{a}_{cm}.$$
- Define your **system** clearly – internal forces cancel, so only external forces change its momentum.
- Exploit symmetry to shortcut a center-of-mass integral.
- Distinguish center of mass from center of gravity (identical in a uniform field).

7

Recap

Unit 2 – key takeaways

1

Center of mass

$x_{cm} = \frac{1}{M} \int x dm$; moved only by external force

2

Newton

$\vec{F}_{net} = m\vec{a}$; third-law pairs act on different bodies

3

Drag

$m dv/dt = mg - bv$; terminal velocity $v_t = mg/b$

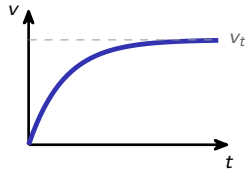
4

Circular

$a_c = v^2/r$ toward the centre; a role, not a new force

Check yourself

- 1 What integral gives the center of mass of a continuous object?
- 2 At terminal velocity, what is the acceleration?
- 3 Two springs in parallel – stiffer or softer than one?
- 4 Rise altitude by one Earth radius – what happens to g ?



Answers 1. $\frac{1}{M} \int x dm$ 2. zero 3. stiffer 4. it drops to one quarter