Force & Translational Dynamics

AP Physics C: Mechanics ▪ Unit 2

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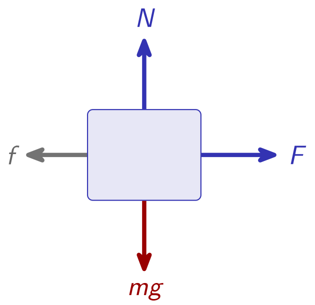
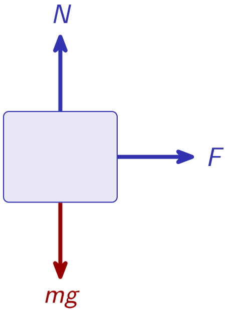


What we will cover

1. Systems and center of mass
2. Free-body diagrams and Newton's laws
3. Gravitation
4. Friction
5. Spring forces
6. Resistive forces and terminal velocity
7. Circular motion



Systems and center of mass

A **system** 系统 moves as if all its mass sat at its **center of mass** 质心:

Discrete and continuous

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1}{M}\int x\, dm$$

Only the **net external force** 净外力 accelerates the center of mass – internal forces cannot.

Worked example – a uniform rod

With $\lambda = M/L$, $dm = \lambda\, dx$:

$$x_{cm} = \frac{1}{M}\int_0^L x\,\lambda\, dx = \frac{L}{2}$$

The integral confirms it balances at the middle.

Free-body diagrams and Newton's laws

Draw every force on a **free-body diagram** 受力图, resolve into components, add to the **net force** 合力.

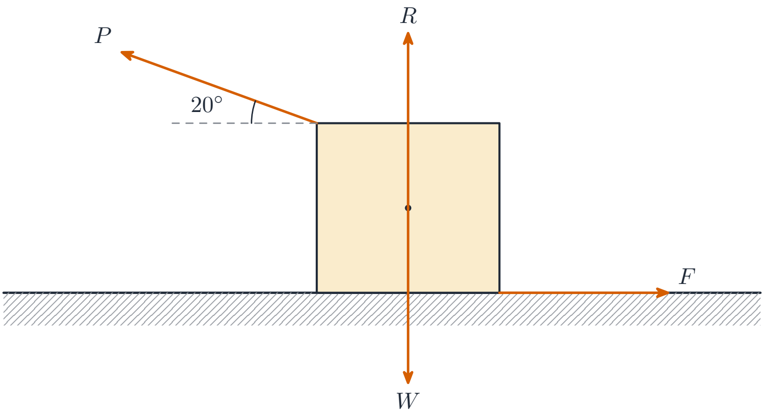
The three laws

1st: inertia $\qquad \vec{F}_{net} = m\vec{a}$

3rd: equal-opposite pairs, different bodies

rocket Force

Free-body diagrams and Newton's laws



Gravitation

Universal gravitation

$$F_g = \frac{Gm_1 m_2}{r^2}, \qquad g = \frac{GM}{r^2}$$

- **mass** 质量 is intrinsic; **weight** 重量 $= mg$
- a sphere pulls as if all mass sat at its centre
- g falls off as $1/r^2$ with **altitude** 海拔

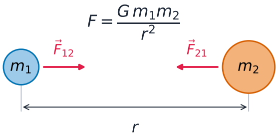
Worked example

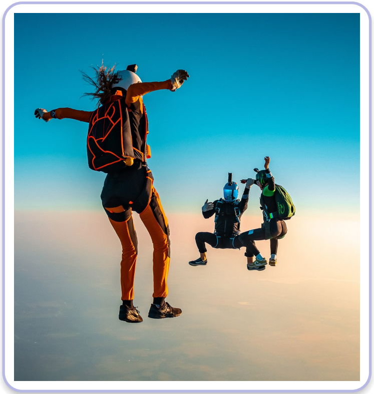
At one Earth radius up, r doubles:

$$g = \frac{9.8}{2^2} = 2.45\,\mathrm{m/s^2}$$

The gentler pull is what lets satellites orbit high up.

Gravitation





skydiver Drag

Friction and spring forces

Friction

$$f_s \leq \mu_s N, \qquad f_k = \mu_k N$$

Depends on N and μ – not on the contact area.

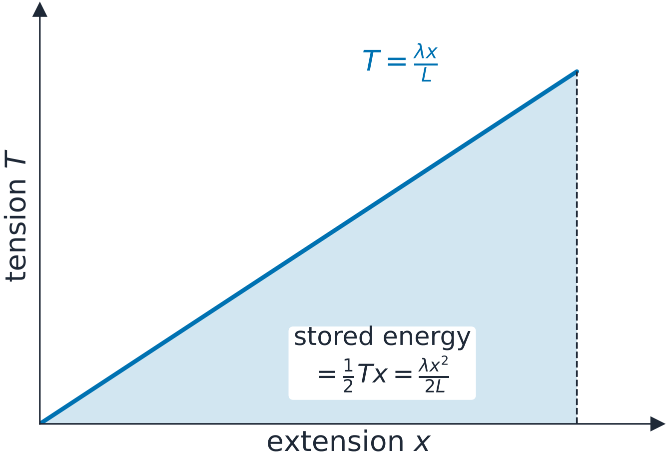
Hooke's law

$$F_s = -kx$$

Combining springs: parallel adds ($k_{eq} = k_1 + k_2$, stiffer); series is softer ($\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$).

Two $100\,\mathrm{N/m}$ springs: parallel 200, series $50\,\mathrm{N/m}$.

Friction and spring forces



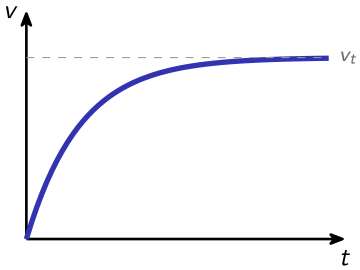
Resistive forces and terminal velocity

Drag grows with speed ($F = -bv$), so Newton's law becomes a **differential equation** 微分方程:

Falling with drag

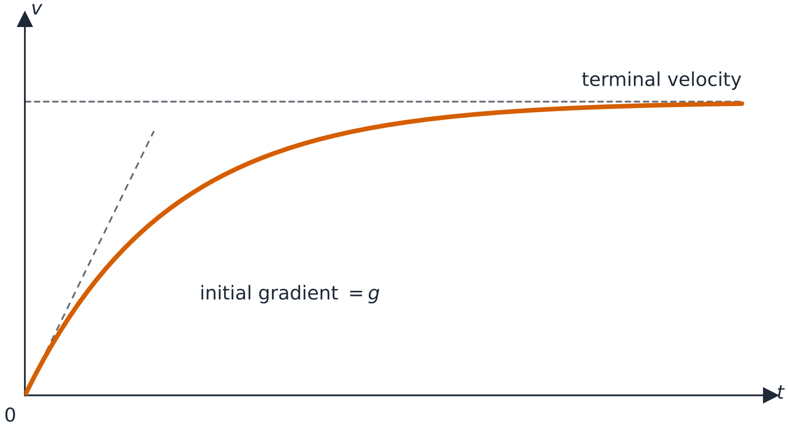
$$m\frac{dv}{dt} = mg - bv$$

At **terminal velocity** 终极速度, $\frac{dv}{dt} = 0$, so $v_t = \frac{mg}{b}$.



v rises smoothly toward v_t and levels off.

Resistive forces and terminal velocity



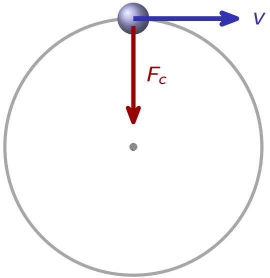
Circular motion

Uniform circular motion 匀速圆周运动: steady speed, changing direction – so it accelerates toward the centre:

Centripetal quantities

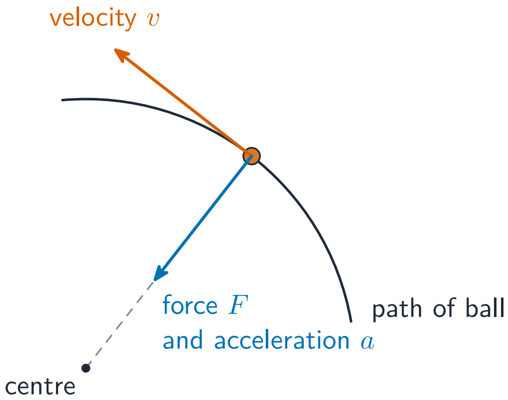
$$a_c = \frac{v^2}{r}, \qquad F_c = \frac{mv^2}{r}$$

Non-uniform motion also has a **tangential** 切向 force that changes the speed.



A car needs F_c from tyre friction – else it skids.

Circular motion



Unit 2 – key takeaways

1 Center of mass

$x_{cm} = \frac{1}{M}\int x\,dm$; moved only by external force

2 Newton

$\vec{F}_{net} = m\vec{a}$; third-law pairs act on different bodies

3 Drag

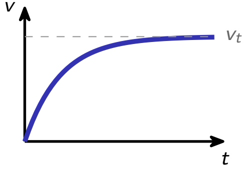
$m\,dv/dt = mg - bv$; terminal velocity $v_t = mg/b$

4 Circular

$a_c = v^2/r$ toward the centre; a role, not a new force

Check yourself

1. What integral gives the center of mass of a continuous object?
2. At terminal velocity, what is the acceleration?
3. Two springs in parallel – stiffer or softer than one?
4. Rise altitude by one Earth radius – what happens to g?



Answers **1.** $\frac{1}{M}\int x\,dm$ **2.** zero **3.** stiffer **4.** it drops to one quarter