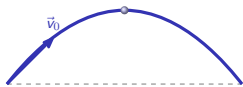


Kinematics

AP Physics C: Mechanics • Unit 1

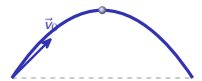
AP Physics C: Mechanics



Kinematics

What we will cover

- 1 Vectors, components, products
- 2 Velocity and acceleration as derivatives
- 3 Motion graphs
- 4 Reference frames and relative motion
- 5 Motion in two dimensions



1 Vectors

Kinematics

↳ Vectors

Vectors and components

Resolve every **vector** 矢量 onto the axes with **unit vectors** 单位矢量 $\hat{i}, \hat{j}, \hat{k}$:

Components

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Add vectors axis by axis to get the **resultant** 合矢量.



Speedometer

Kinematics

↳ Vectors

Two ways to multiply vectors

Dot product → a scalar

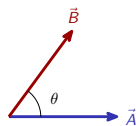
$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Work is defined this way.

Cross product → a vector

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

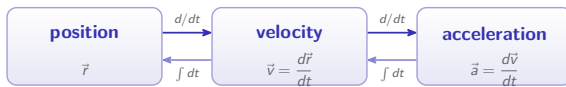
⊥ to both (right-hand rule). Torque and angular momentum use this.



2 Derivatives

Velocity and acceleration are derivatives

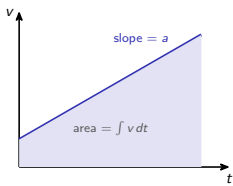
Differentiate to move **down** the chain; integrate to move **back up**:



So $\vec{a} = \frac{d^2 \vec{r}}{dt^2}$, and running it backwards: $\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a} dt'$ – valid even when a is **not** constant.

3 Motion graphs

Motion graphs: slope and area



$x-t$: slope = $v = \frac{dx}{dt}$
 $v-t$: slope = a ; area = $\int v dt = \Delta x$.

Constant a (a special case)

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2} at^2$$

4 Relative motion

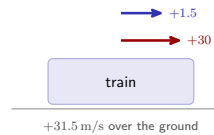
Reference frames and relative motion

Velocities add as vectors between frames:

Frame conversion

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

Frames moving at **constant** relative velocity share the same acceleration – they are **inertial** 惯性系, and Newton's laws hold identically.



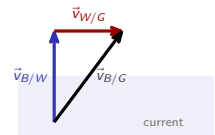
A boat crossing a river

The boat's ground velocity is the **vector sum** of its velocity through the water and the current:

$$\vec{v}_{B/G} = \vec{v}_{B/W} + \vec{v}_{W/G}$$

Across 3.0, downstream 2.0

$$|\vec{v}_{B/G}| = \sqrt{3.0^2 + 2.0^2} \approx 3.6 \text{ m/s}$$



5 Projectiles

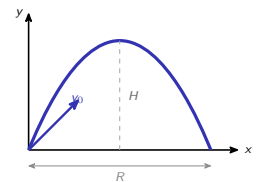
Projectile motion

The axes are **independent**, joined only by time t :

- horizontal: $a_x = 0$, so v_x constant
- vertical: $a_y = -g$ (free fall)

Eliminating t makes y a quadratic in x – the **trajectory** 轨迹 is a **parabola** 抛物线.

A dropped and a launched ball land together.



Projectile formulas

Fired from the ground at speed v_0 , angle θ :

Three key results

$$t_{\text{flight}} = \frac{2v_0 \sin \theta}{g}$$

$$H = \frac{v_0^2 \sin^2 \theta}{2g}, \quad R = \frac{v_0^2 \sin 2\theta}{g}$$

Range 射程 is greatest at $\theta = 45^\circ$; angles like 30° and 60° land at the **same** distance.

Uniform circular motion 匀速圆周运动:
 $a_c = \frac{v^2}{r}$ toward the centre – a preview of dynamics.

6 Recap

Unit 1 – key takeaways

1 Vectors

components $\hat{i}, \hat{j}, \hat{k}$; dot $\rightarrow$ scalar, cross $\rightarrow$ vector

2 Calculus

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$; integrate to reverse

3 Relative

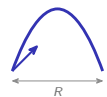
$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$; inertial frames

4 Projectiles

independent axes; range peaks at 45°

Check yourself

- What operation takes velocity back to position?
- Dot product or cross product – which gives a scalar?
- Two frames move at constant relative velocity – do they measure the same acceleration?
- Which launch angle gives the greatest range?



Answers 1. integration 2. the dot product 3. yes 4. 45°