Kinematics

AP Physics C: Mechanics ▪ Unit 1

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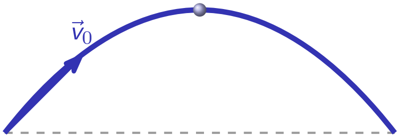
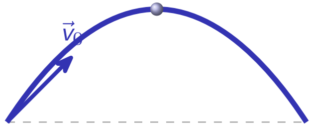


What we will cover

1. Vectors, components, products
2. Velocity and acceleration as derivatives
3. Motion graphs
4. Reference frames and relative motion
5. Motion in two dimensions



Vectors and components

Resolve every **vector** 矢量 onto the axes with **unit vectors** 单位矢量 $\hat{i}, \hat{j}, \hat{k}$:

Components

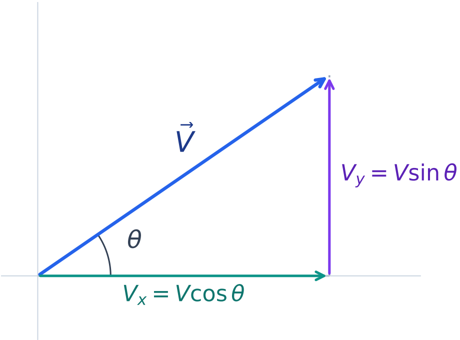
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Add vectors axis by axis to get the **resultant** 合矢量.

speedometer

Vectors and components



train Kinematics

Two ways to multiply vectors

Dot product $\rightarrow$ a *scalar*

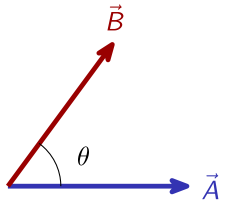
$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

Work is defined this way.

Cross product $\rightarrow$ a *vector*

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

$\perp$ to both (right-hand rule). Torque and angular momentum use this.



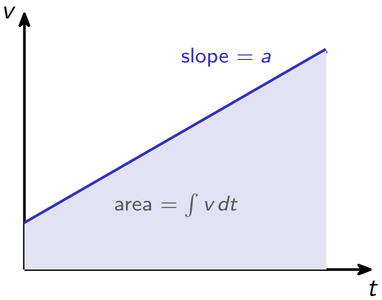
Velocity and acceleration are derivatives

Differentiate to move down the chain; integrate to move back up:

$$\begin{tikzcd}[column sep=huge] \begin{array}{c}\textbf{position}\\ \vec{r}\end{array} \arrow[r, shift left, "d/dt"] & \begin{array}{c}\textbf{velocity}\\ \vec{v} = \dfrac{d\vec{r}}{dt}\end{array} \arrow[l, shift left, "\int dt"] \arrow[r, shift left, "d/dt"] & \begin{array}{c}\textbf{acceleration}\\ \vec{a} = \dfrac{d\vec{v}}{dt}\end{array} \arrow[l, shift left, "\int dt"] \end{tikzcd}$$

So $\vec{a} = \dfrac{d^2\vec{r}}{dt^2}$, and running it backwards: $\vec{v}(t) = \vec{v}_0 + \displaystyle\int_0^t \vec{a}\, dt'$ – valid even when a is not constant.

Motion graphs: slope and area



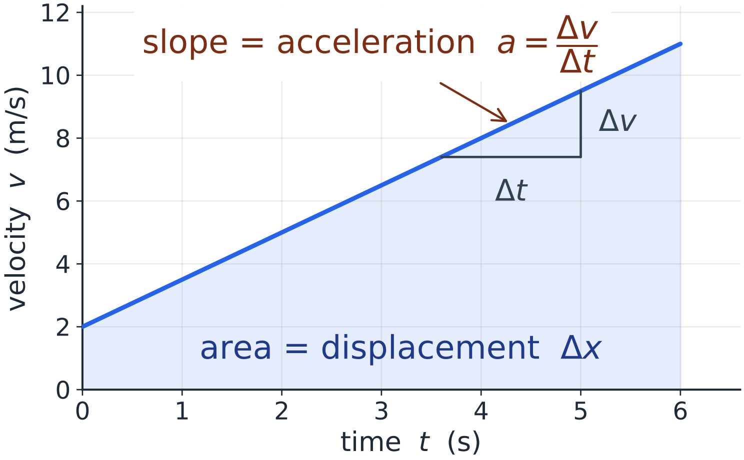
x–t: slope $= v = \dfrac{dx}{dt}$.

v–t: slope $= a$; area $= \displaystyle\int v\,dt = \Delta x$.

Constant a (a special case)

$$v = v_0 + at, \;\; x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

Motion graphs: slope and area



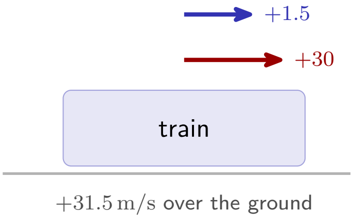
Reference frames and relative motion

Velocities add as vectors between frames:

Frame conversion

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

Frames moving at constant relative velocity share the same acceleration – they are **inertial** 惯性系, and Newton's laws hold identically.



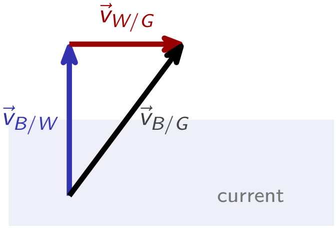
A boat crossing a river

The boat's ground velocity is the vector sum of its velocity through the water and the current:

$$\vec{v}_{B/G} = \vec{v}_{B/W} + \vec{v}_{W/G}$$

Across 3.0, downstream 2.0

$$|\vec{v}_{B/G}| = \sqrt{3.0^2 + 2.0^2} \approx 3.6\,\mathrm{m/s}$$



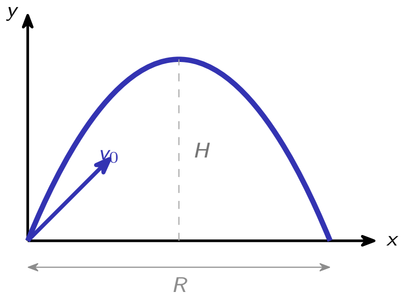
Projectile motion

The axes are independent, joined only by time t:

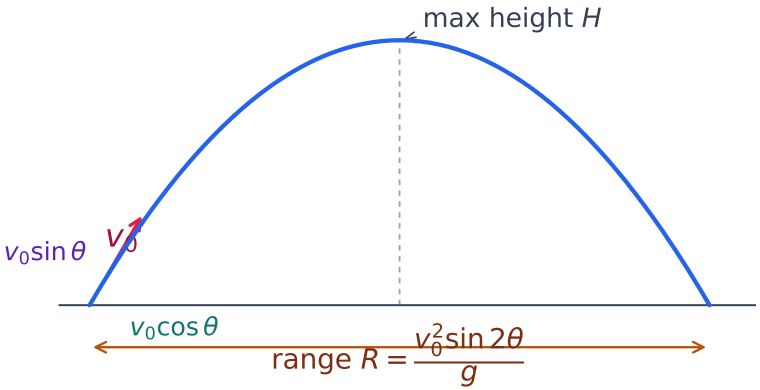
- horizontal: $a_x = 0$, so v_x constant
- vertical: $a_y = -g$ (free fall)

Eliminating t makes y a quadratic in x – the **trajectory** 轨迹 is a **parabola** 抛物线.

> A dropped and a launched ball land together.



Projectile motion



Projectile formulas

Fired from the ground at speed v_0, angle θ:

Three key results

$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g}$$

$$H = \frac{v_0^2 \sin^2\theta}{2g}, \qquad R = \frac{v_0^2 \sin 2\theta}{g}$$

Range 射程 is greatest at $\theta = 45^\circ$; angles like 30° and 60° land at the same distance.

Uniform circular motion 匀速圆周运动:
$a_c = \dfrac{v^2}{r}$ toward the centre – a preview of dynamics.

Unit 1 – key takeaways

1 Vectors

components $\hat{i}, \hat{j}, \hat{k}$; dot $\to$ scalar, cross $\to$ vector

2 Calculus

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$; integrate to reverse

3 Relative

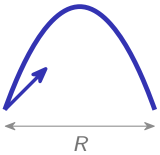
$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$; inertial frames

4 Projectiles

independent axes; range peaks at $45°$

Check yourself

1. What operation takes velocity back to position?
2. Dot product or cross product – which gives a scalar?
3. Two frames move at constant relative velocity – do they measure the same acceleration?
4. Which launch angle gives the greatest range?



Answers **1.** integration **2.** the dot product **3.** yes **4.** 45°