Physics C rubrics award the symbolic expression, so every solution here is carried through in symbols and only evaluated at the end. Where a limiting-case check is available it is shown, because examiners award that reasoning explicitly. 本册先用符号推导、最后代入数值，并在可行处做极限检验。

Motion with variable acceleration

A particle moves along the x-axis with acceleration $a = -kv$, where k is a positive constant and the initial speed is v_0. Find $v(t)$ and the total distance travelled before it stops.

$v = v_0 e^\wedge(-kt)$; total distance v_0/k.

Acceleration depends on velocity, so suvat is illegal – say so. Write $dv/dt = -kv$ and separate: $dv/v = -k\ dt$. Integrating from v_0 to v and 0 to t gives $ln(v/v_0) = -kt$, so $v = v_0 e^\wedge(-kt)$. For the distance, integrate the speed from 0 to infinity: $x = \ v_0 e^\wedge(-kt)dt = v_0/k$. Check the limits: a larger k means stronger drag and a shorter distance, which the expression shows, and the particle never formally stops in finite time but travels a finite distance – a result worth stating.

A block slides down a frictionless incline of angle θ from rest through a height h. Find its speed at the bottom, and then repeat with a coefficient of kinetic friction μ.

Frictionless: $v = \surd(2gh)$. With friction: $v = \surd(2gh(1 - \mu \cot \theta))$.

Frictionless case by energy: $mgh = ½mv^2$, so $v = \surd(2gh)$ – independent of both the mass and the angle. With friction, the normal force is $mg \cos \theta$, so the friction force is $\mu mg \cos \theta$ acting over the slope length $h/\sin \theta$. Energy: $mgh - \mu mg \cos \theta\ (h/\sin \theta) = ½mv^2$, giving $v = \surd(2gh(1 - \mu \cot \theta))$. Check the limits: $\mu = 0$ recovers the first result, and the speed reaches zero when $\mu = \tan \theta$, which is exactly the condition for the block not to slide at all. Presenting that check is a scored line.

Rotation

A uniform solid cylinder of mass M and radius R rolls without slipping down an incline of angle θ. Find its acceleration.

$a = (2/3)g \sin \theta$.

Two routes; energy is faster. Rolling kinetic energy is $½Mv^2 + ½I\omega^2$, and for a solid cylinder $I = ½MR^2$. With rolling without slipping, $\omega = v/R$, so the rotational term is $¼Mv^2$ and the total is $¾Mv^2$. Setting $Mgd \sin \theta = ¾Mv^2$ and differentiating, or using $v^2 = 2ad$, gives $a = (2/3)g \sin \theta$. Check: it is less than $g \sin \theta$, as it must be, because some of the energy goes into rotation; and it is independent of M and R, which is why all solid cylinders roll down together.

A uniform rod of mass M and length L is pivoted at one end and released from horizontal. Find its angular speed when vertical.

$\omega = \surd(3g/L)$.

The moment of inertia of a rod about its end is $ML^2/3$ – either quote it or derive it from $\ r^2 dm$ with $dm = (M/L)dr$. The centre of mass falls L/2, so energy conservation gives $Mg(L/2) = ½(ML^2/3)\omega^2$. Solving: $\omega^2 = 3g/L$, so $\omega = \surd(3g/L)$. Note that using the rod's centre-of-mass speed with $½Mv^2$ alone would be wrong, since the rod rotates rather than translating – that is the error this question is set to catch.

Oscillation and gravitation

A block of mass m on a frictionless surface is attached to two springs of constants k_1 and k_2, one on each side. Show the motion is simple harmonic and find the period.

$T = 2\pi\sqrt{(m/(k_1 + k_2))}$.

Displace the block by x. One spring is compressed and the other stretched, but BOTH push it back, so the restoring force is $-(k_1 + k_2)x$. Newton's second law gives $m\, d^2x/dt^2 = -(k_1 + k_2)x$, which is exactly the form $d^2x/dt^2 = -\omega^2 x$ with $\omega^2 = (k_1 + k_2)/m$ – that identification is the required demonstration that the motion is simple harmonic, and it is a scored statement. The period follows as $T = 2\pi/\omega = 2\pi\sqrt{(m/(k_1 + k_2))}$. Check: two identical springs give an effective constant of 2k, so the block oscillates faster than with one, as expected.

A satellite of mass m orbits a planet of mass M in a circular orbit of radius r. Derive the total mechanical energy, and hence the energy needed to move it to radius 2r.

$E = -GMm/2r$; the energy required is $GMm/4r$.

For a circular orbit, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$, so $½mv^2 = GMm/2r$. The potential energy is $-GMm/r$, so the total is $E = GMm/2r - GMm/r = -GMm/2r$. At radius 2r the total is $-GMm/4r$. The energy that must be supplied is the difference, $(-GMm/4r) - (-GMm/2r) = +GMm/4r$. Note that the kinetic energy DECREASES as the orbit rises even though energy was added – a counter-intuitive result the exam asks about directly, so state it.