Course Companion

AP Physics C: Mechanics

Every topic handout in one volume
全部专题讲义合集

exercises.lol

Contents 目录

Kinematics

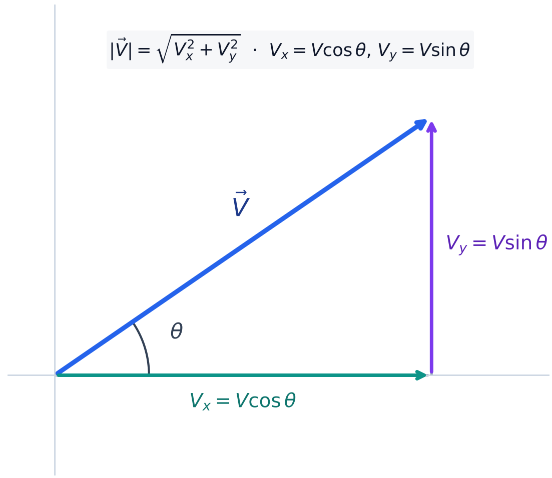
AP Physics C: Mechanics

Scalars and Vectors

Kinematics 运动学 describes motion. Two kinds of quantity:

- A **scalar** 标量 has only size (**magnitude** 大小): distance, speed, mass, time.
- A **vector** 矢量 has magnitude **and** direction: displacement, velocity, acceleration, force.

In this calculus-based course you resolve vectors into **components** 分量 and add them component by component; a vector's magnitude is $\sqrt{v_x^2 + v_y^2 + \cdots}$. Physics C also writes vectors with **unit vectors** 单位矢量: $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are directions of length one along x and y – handy because calculus can then act on each component separately.



Resolving a vector into its x and y components

A car speedometer: speed is the magnitude of velocity — a scalar rate of motion

Displacement, Velocity, and Acceleration

Because motion can change continuously, define the rates as **derivatives** 导数:

$$v = \frac{dx}{dt}, \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

Reversing this, integrate to recover velocity and position:

$$v(t) = v_0 + \int_0^t a\,dt, \qquad x(t) = x_0 + \int_0^t v\,dt.$$

Distinguish the *average* from the *instantaneous*: **average velocity** 平均速度 is $\Delta x/\Delta t$ over an interval, while **instantaneous velocity** 瞬时速度 is the derivative at one moment – the slope of the tangent line, and what a speedometer shows.

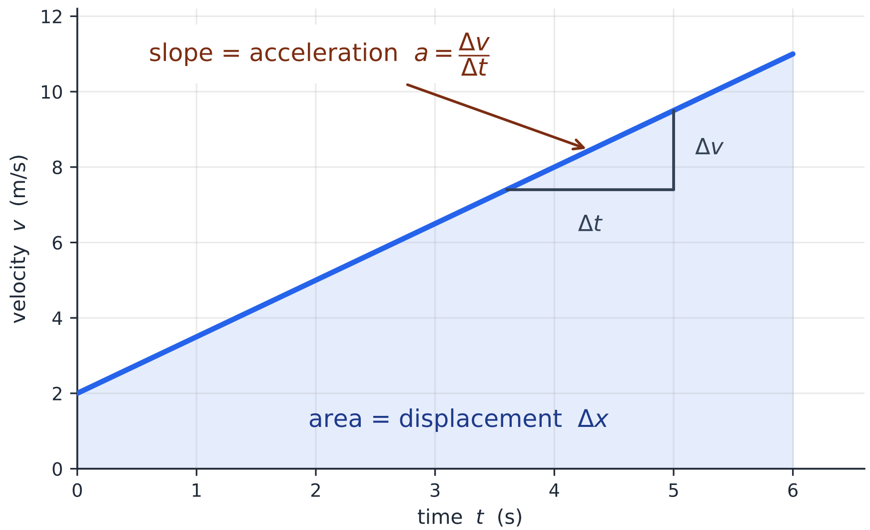
For **constant** acceleration these give the familiar kinematic equations ($v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\,\Delta x$). The most important constant-a case is **free fall** 自由落体: near Earth's surface every object, heavy or light, accelerates downward at $g = 9.8\ \mathrm{m/s^2}$ once air resistance is negligible. When a or v varies with time, use the integrals directly.

Worked example. A particle moves with $x(t) = 2t^3 - 3t^2$ (metres). Differentiate for velocity and acceleration:

$$v = \frac{dx}{dt} = 6t^2 - 6t, \qquad a = \frac{dv}{dt} = 12t - 6.$$

At $t = 2$ s, $v = 24 - 12 = 12$ m/s and $a = 24 - 6 = 18\ \mathrm{m/s^2}$. This calculus link – not just the constant-a formulas – is what distinguishes Physics C.

Worked example. A particle starts from rest at the origin with $a(t) = 6t$. Integrate: $v = \int 6t\,dt = 3t^2$ and $x = \int 3t^2\,dt = t^3$. At $t = 2$ s, $v = 12$ m/s and $x = 8$ m.

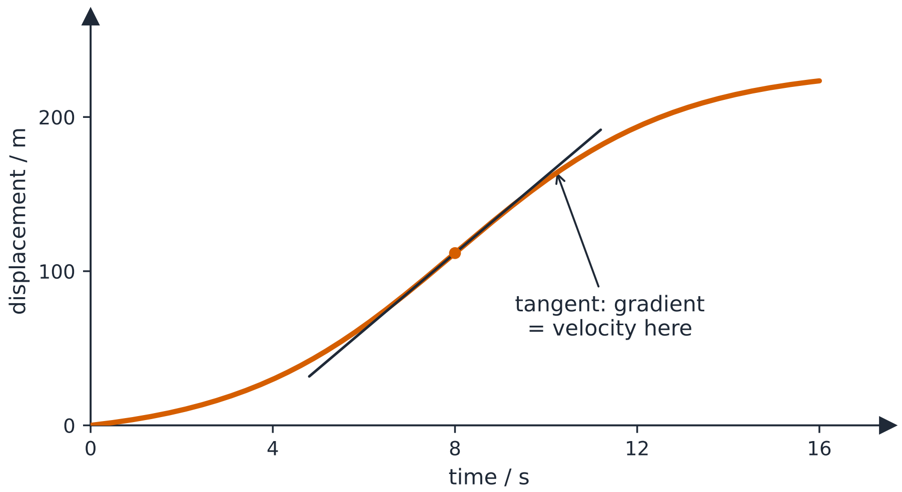


On a velocity-time graph the gradient is the acceleration and the area is the displacement

A high-speed train: kinematics describes displacement, velocity and acceleration over time

Representing Motion

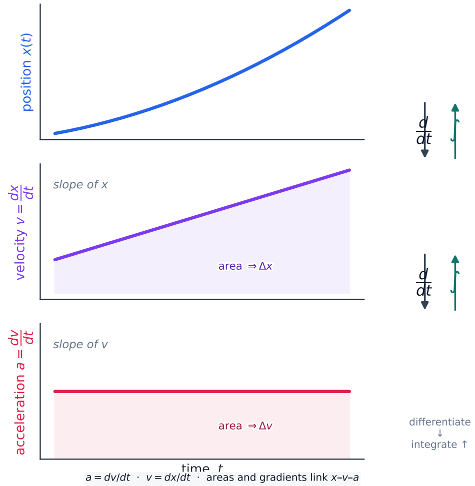
Move fluently between description, graph, table, and equation:



A displacement-time graph: the slope at any instant is the velocity

- On a **position–time** graph, the slope (dx/dt) is velocity.
- On a **velocity–time** graph, the slope is acceleration and the **area** ($\int v\,dt$) is displacement.
- On an **acceleration–time** graph, the area ($\int a\,dt$) is the change in velocity.

Slopes are derivatives; areas are integrals – the two are inverse operations here.



Position, velocity, and acceleration are linked by differentiation

Reference Frames and Relative Motion

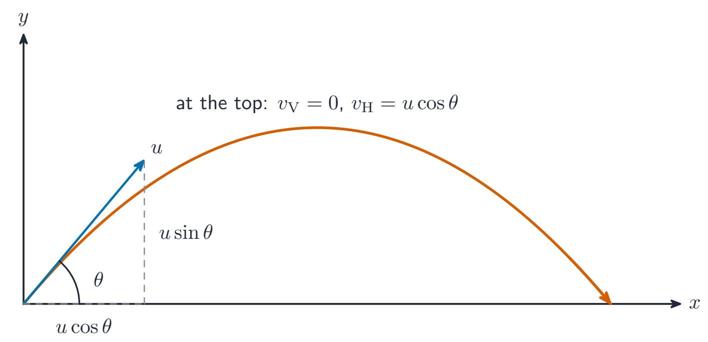
All motion is relative to a **reference frame** 参考系. Combine velocities by vector addition: $\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$. This handles boats crossing rivers and passengers on moving vehicles – **relative motion** 相对运动 problems. Read the subscripts as a chain: "A relative to B" plus "B relative to C" gives "A relative to C".

Worked example. A boat that moves at 4.0 m/s in still water heads straight across a river flowing at 3.0 m/s. Relative to the ground the boat moves at $\sqrt{4.0^2 + 3.0^2} = 5.0$ m/s, angled downstream. If the river is 80 m wide, the crossing still takes $t = \frac{80}{4.0} = 20$ s – only the across-stream component crosses the river; the current just carries the boat 60 m downstream.

Motion in Two or Three Dimensions

In two or three dimensions, position is a vector $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$, and velocity and acceleration are its successive derivatives – each component handled independently.

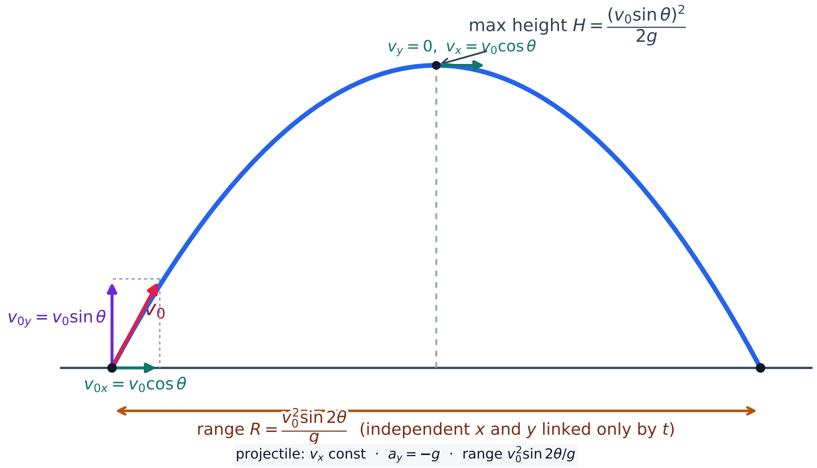
Worked example. $\vec{r}(t) = \left(2t^2\right)\hat{i} + \left(4t - t^3\right)\hat{j}$ (metres). Differentiating each component: $\vec{v} = 4t\,\hat{i} + (4 - 3t^2)\,\hat{j}$ and $\vec{a} = 4\,\hat{i} - 6t\,\hat{j}$. At $t = 1$ s: $\vec{v} = 4\hat{i} + 1\hat{j}$, so the speed is $\sqrt{17} \approx 4.1$ m/s – no new physics, just one derivative per component.



A projectile launched at an angle: the horizontal and vertical motions are independent

For **projectile motion** 抛体运动, horizontal and vertical motions are independent, linked only by time t: horizontally $a_x = 0$ (constant velocity), vertically $a_y = -g$. The path is a parabola. This component method extends to any two-dimensional motion where the accelerations along each axis are known.

Worked example. A ball is launched at 20 m/s, 30° above the horizontal ($g = 9.8$ m/s^2). The components are $v_{0x} = 20\cos 30° = 17.3$ m/s and $v_{0y} = 20\sin 30° = 10$ m/s. The vertical motion sets the time: total flight $= \dfrac{2v_{0y}}{g} = \dfrac{20}{9.8} = 2.0$ s, maximum height $= \dfrac{v_{0y}^2}{2g} = 5.1$ m, and range $= v_{0x} \times 2.0 = 35$ m.



A projectile launched at an angle: velocity components, maximum height, and range

Exam tips

- Resolve every vector into components before adding — never add magnitudes at an angle directly.
- On Physics C you are expected to use **calculus**: velocity is $\vec{v} = \frac{d\vec{r}}{dt}$ and acceleration $\vec{a} = \frac{d\vec{v}}{dt}$; reverse with integration.
- Carry and check **units** and treat direction with signs (choose a positive axis and stick to it).
- Use the dot product for work-type quantities and the cross product for torque and angular momentum.
- Sketch the vectors — a diagram catches sign and direction errors the algebra hides.

Force and Translational Dynamics

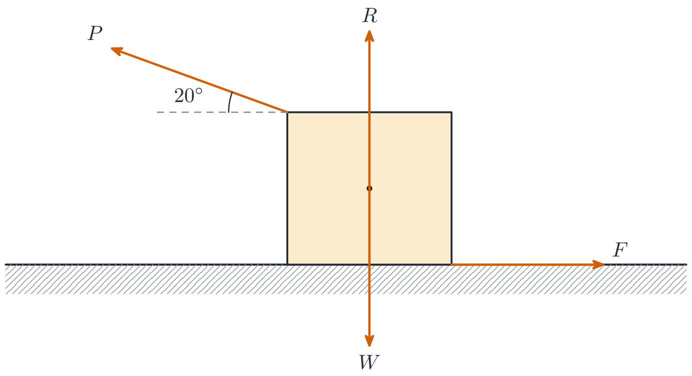
AP Physics C: Mechanics

Systems and Center of Mass

A **system** 系统 is the object or objects you analyze, treated as a point at its **center of mass** 质心. For a set of masses, $x_{\text{cm}} = \dfrac{\sum m_i x_i}{\sum m_i}$; for a continuous body, $x_{\text{cm}} = \dfrac{1}{M}\int x\,dm$. Only **external** forces move the center of mass.

Forces and Free-Body Diagrams

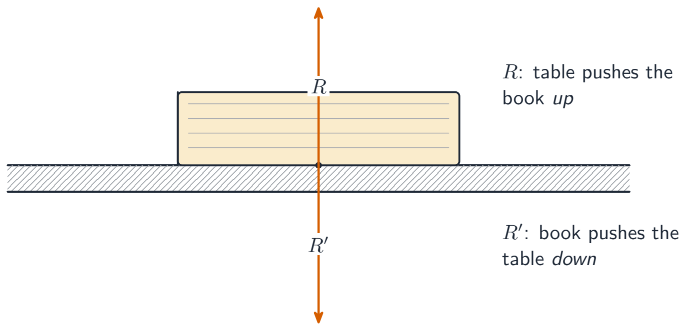
A **force** 力 is a push or pull (a vector, in newtons). A **free-body diagram** 受力图 draws one object with an arrow for **every** force on it – weight, normal, tension, friction, applied, drag. Draw it first; it sets up every dynamics equation.



A free-body diagram shows every force acting on one object

Newton's Third Law

Newton's third law 牛顿第三定律: A pushes B, B pushes back equally and oppositely. The pair acts on **different** objects, so it never cancels within one free-body diagram.



A Newton's third-law pair: equal and opposite forces on two different objects

Newton's First Law

Newton's first law (**inertia** 惯性): with zero **net force** 合力, velocity stays constant – the object is in **translational equilibrium** 平动平衡.

Newton's Second Law

The general form uses momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \quad \text{(for constant mass).}$$

Apply it one axis at a time. When forces depend on velocity or position, this becomes a **differential equation** 微分方程 to solve.

Worked example. A 2.0 kg block slides down a frictionless **incline** 斜面 at 30°. Only the along-ramp component of gravity drives it, so $a = g\sin 30° = 9.8(0.5) = 4.9\ \text{m/s}^2$, independent of the mass.

Worked example (two bodies). A 6.0 kg cart on a frictionless table is pulled by a string over a light pulley to a hanging 2.0 kg mass. Treat the pair as one system: only the hanging weight drives it, so $a = \dfrac{2.0(9.8)}{8.0} = 2.45\ \text{m/s}^2$. Then isolate the cart alone to find the string **tension** 张力: $T = 6.0(2.45) \approx 15$ N. System first for a, single body for internal forces – that two-step is the standard pattern.

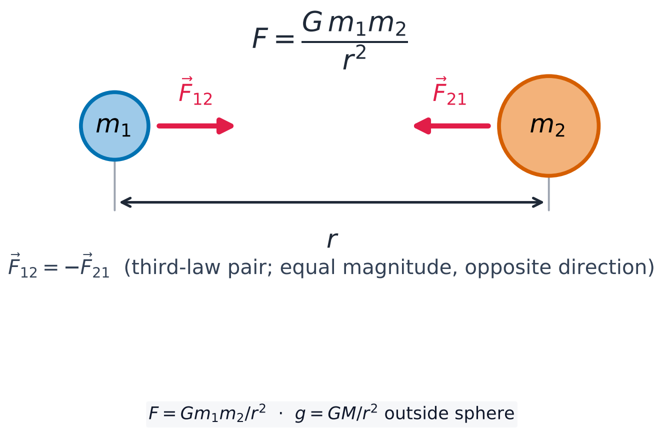
A rocket launch: net force changes momentum — F_net = ma for translational dynamics

Gravitational Force

Near a surface, weight is $F_g = mg$. In general, **Newton's law of gravitation** 万有引力定律:

$$F_g = \frac{Gm_1m_2}{r^2},$$

attractive and inverse-square. The gravitational field is $g = \frac{GM}{r^2}$.



Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

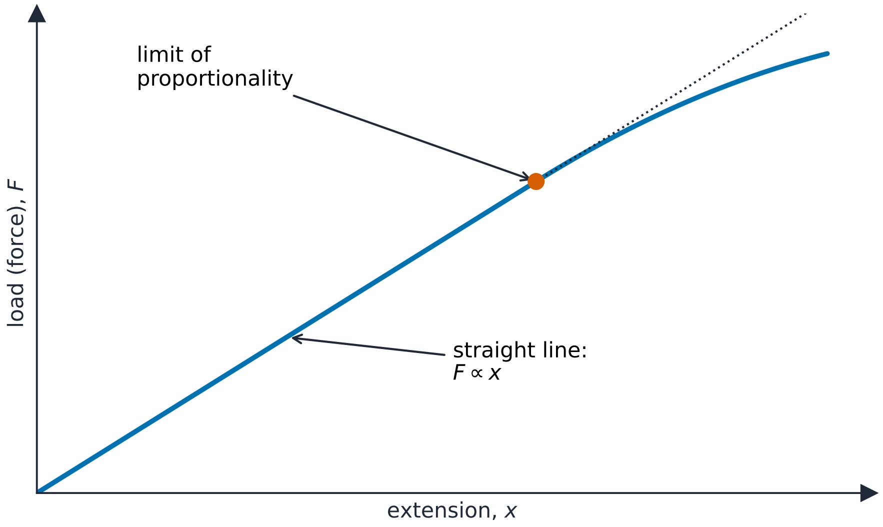
Kinetic and Static Friction

Friction 摩擦力 opposes sliding along a surface: **kinetic** 动摩擦 $f_k = \mu_k N$ while sliding, and **static** 静摩擦 $f_s \leq \mu_s N$ up to a maximum before sliding. N is the **normal force** 法向力. Note the inequality: static friction is only as large as it needs to be, and $\mu_s N$ is its *ceiling*, not its value.

Worked example. The same block on the same 30° incline, now with $\mu_k = 0.20$. Perpendicular to the ramp: $N = mg\cos 30°$. Along the ramp: $ma = mg\sin 30° - \mu_k mg\cos 30°$, so $a = g(\sin 30° - 0.20\cos 30°) = 9.8(0.500 - 0.173) = 3.2$ m/s^2 – the mass cancels again.

Spring Forces

An ideal spring obeys **Hooke's law** 胡克定律 $F_s = -kx$, a restoring force set by the **spring constant** 弹簧劲度系数 k. Its stored energy is $U = \frac{1}{2}kx^2$.



Hooke's law: extension is proportional to load up to the limit of proportionality

Combined springs act as one **equivalent spring** of constant k_{eq}. In **parallel** (side by side, sharing the load) the constants add, $k_{\text{eq}} = k_1 + k_2$ – stiffer than either. In **series** (end to end) they combine as $\dfrac{1}{k_{\text{eq}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$ – softer than the softest one. Two springs pulling a mass from opposite sides also act in parallel, giving a restoring constant $k_1 + k_2$. Whichever you have, the SHM period uses k_{eq}: $T = 2\pi\sqrt{m/k_{\text{eq}}}$.

Resistive Forces

A **resistive force** 阻力 (drag) opposes motion through a fluid and **grows with speed**, often modeled as $F = -bv$ or $F = -cv^2$. Newton's second law then gives a differential equation, e.g. $m\dfrac{dv}{dt} = mg - bv$ for a falling object. As speed rises, drag builds until it balances the driving force; the object then stops accelerating and moves at a constant **terminal velocity** 终极速度, found by setting the net force (and dv/dt) to zero.

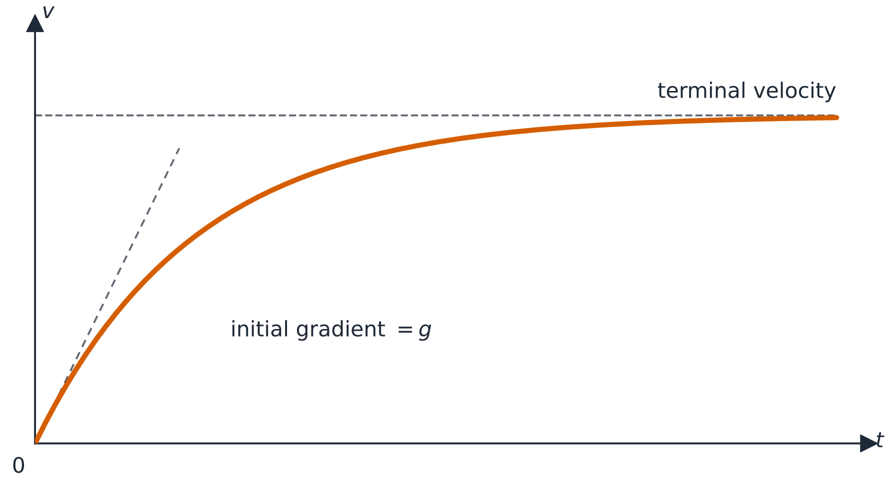
Worked example. For a falling object with $m\dfrac{dv}{dt} = mg - bv$, the terminal velocity

is where $\dfrac{dv}{dt} = 0$: $mg = bv_T$, so $v_T = \dfrac{mg}{b}$. With $m = 0.10$ kg and $b = 0.50$ kg/s, $v_T = \dfrac{0.10(9.8)}{0.50} = 2.0$ m/s.

To get the full motion $v(t)$, **separate the variables** 分离变量 and integrate from rest:

$$\int_0^v \frac{dv'}{mg - bv'} = \int_0^t \frac{dt'}{m} \Rightarrow -\frac{1}{b}\ln\frac{mg - bv}{mg} = \frac{t}{m},$$

which rearranges to $v(t) = v_T\left(1 - e^{-bt/m}\right)$ – the speed rising exponentially toward v_T. Showing this integration is the calculus skill a Physics C free-response rewards, not just quoting the final formula.



An object falling through a fluid speeds up to a terminal velocity

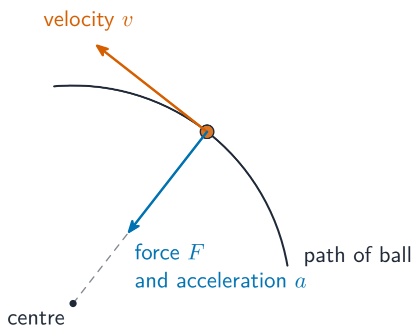
Skydivers in freefall: drag grows with speed until it balances weight at terminal velocity

Circular Motion

Uniform circular motion has a **centripetal acceleration** 向心加速度 $a_c = \frac{v^2}{r}$ pointing to the center, requiring a **net inward force** $F_c = \frac{mv^2}{r}$ supplied by a real force (tension, gravity, friction, normal). There is no outward force. For vertical circles and banked curves, decompose the real forces to find which provides the centripetal requirement.

Worked example. A 0.50 kg ball is whirled on a 1.0 m string in a horizontal circle at 3.0 m/s. The string tension supplies the whole centripetal force: $T = \frac{mv^2}{r} = \frac{0.50(3.0)^2}{1.0} = 4.5$ N.

Worked example (vertical circle). At the top of a vertical loop of radius r, gravity and the normal force *both* point toward the center: $N + mg = \frac{mv^2}{r}$. The slowest possible speed at the top is where the track pushes with nothing at all ($N = 0$): $v_{\min} = \sqrt{gr}$. For $r = 2.5$ m: $v_{\min} = \sqrt{9.8(2.5)} = 4.9$ m/s. Any slower and the cart leaves the track before the top.



The velocity points along the tangent; the centripetal force points to the centre

Exam tips

- Locate the **center of mass** with $x_{cm} = \frac{1}{M}\int x\,dm$ (or $\frac{\sum m_i x_i}{\sum m_i}$ for point masses) and use λ, σ, ρ for the mass element.
- The net external force moves the center of mass as if all mass sat there: $\vec{F}_{net} = M\vec{a}_{cm}$.

- Define your **system** clearly — internal forces cancel, so only external forces change its momentum.
- Exploit symmetry to shortcut a center-of-mass integral.
- Distinguish center of mass from center of gravity (identical in a uniform field).

Work, Energy, and Power

AP Physics C: Mechanics

Translational Kinetic Energy

Kinetic energy 动能 is the energy of motion, a **scalar** 标量 measured in **joules** 焦耳 (J):

$$K = \tfrac{1}{2}mv^2.$$

It grows with the *square* of speed – doubling the speed quadruples K. One subtlety worth knowing: kinetic energy depends on the observer's **reference frame** 参考系. A passenger walking down a train has a small K measured inside the train and a huge one measured from the ground – both observers are right, each in their own frame.

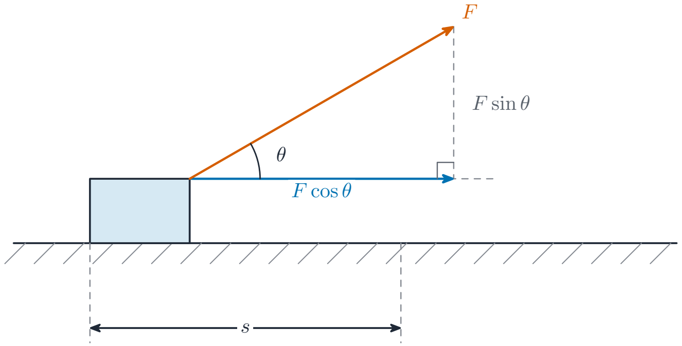
Work

Work 功 is energy transferred into or out of a system by a force acting over a distance. For a variable force it is an **integral** 积分 along the path:

$$W = \int_a^b \vec{F} \cdot d\vec{r} = \int F \cos\theta \, dr.$$

For a constant force this reduces to $W = Fd\cos\theta$; on a force–position graph, work is the **area** under the curve. Work is a scalar with a sign, and the sign is physics, not bookkeeping:

- **Positive** – force has a component along the motion (it speeds the object up).
- **Negative** – force opposes the motion (**friction** 摩擦力 and air drag do negative work).
- **Zero** – force perpendicular to the motion (the **normal force** 法向力 on a sliding block, gravity on a horizontal move, tension in a circular swing).



Only the force component along the displacement does work

The **work–energy theorem 动能定理** collects every force's contribution: the *net* work equals the change in kinetic energy,

$$W_{\text{net}} = \sum W_i = \Delta K.$$

Worked example. A variable force $F(x) = 3x^2$ N (along the motion) acts from $x = 0$ to $x = 2$ m: $W = \int_0^2 3x^2\,dx = \left[x^3\right]_0^2 = 8$ J. Acting alone on a body starting from rest, it would raise the kinetic energy to exactly 8 J.

Potential Energy

Potential energy 势能 is energy a *system* stores by the positions of its parts – it exists only for **conservative forces 保守力**, whose work is path independent. It is defined through work:

$$\Delta U = -\int_a^b \vec{F}\cdot d\vec{r},$$

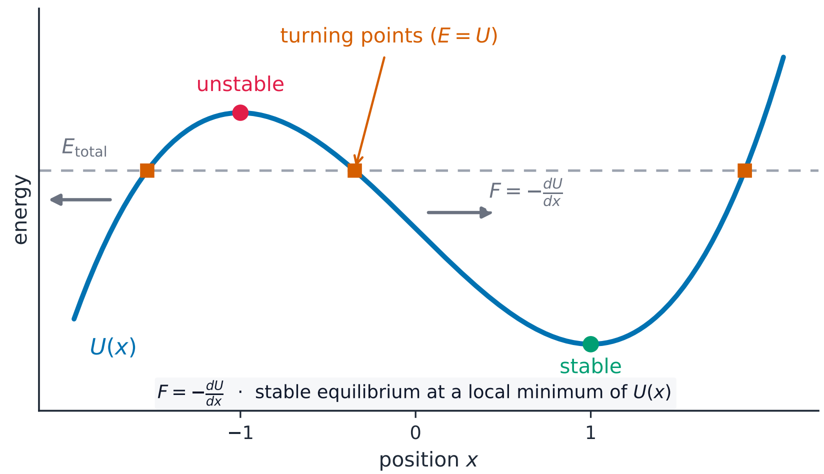
and you are free to *choose where* $U = 0$ – only changes in U matter, so pick the zero that makes the problem simplest. The standard results:

- Gravity near a surface: $\Delta U_g = mg\,\Delta y$.
- Gravity in general: $U_g = -\dfrac{Gm_1m_2}{r}$ (zero at infinite separation).
- Spring: $U_s = \frac{1}{2}k(\Delta x)^2$, with Δx measured from natural length.

For systems of several objects, add the potential energy of each *pair*. Turning the definition around, a conservative force is minus the **derivative 导数** of its potential energy:

$$F_x = -\frac{dU}{dx}.$$

The force points "downhill" on the $U(x)$ curve. Equilibrium sits where the slope is zero: a minimum is a **stable equilibrium 稳定平衡** (displaced, the force pushes back), a maximum is an **unstable equilibrium 不稳定平衡** (displaced, the force pushes away).



On a potential-energy curve the force points downhill and $E = U$ marks the turning points

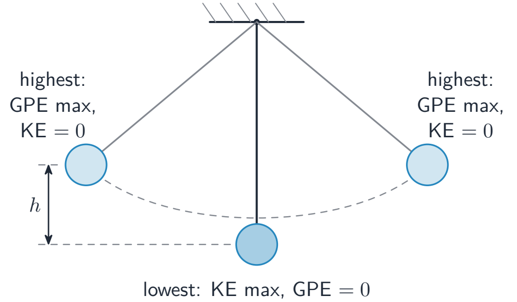
Worked example. Given $U(x) = 2x^3 - 6x$ (joules), the force is $F = -\dfrac{dU}{dx} = 6 - 6x^2$. Equilibria sit at $F = 0$: $x = \pm 1$. Since $\dfrac{d^2U}{dx^2} = 12x$ is positive at $x = +1$ (a minimum – stable) and negative at $x = -1$ (a maximum – unstable), the two points behave oppositely.

Conservation of Energy

Energy is conserved in *all* interactions – the question is only where it goes. **Mechanical energy 机械能** is the sum $E = K + U$. If the external work on a system is zero and nothing inside it acts through **nonconservative forces 非保守力**, then

$$K_1 + U_1 = K_2 + U_2.$$

When friction or drag act, they convert mechanical energy into **thermal energy 热能** ($\Delta E_{\text{mech}} = -F_f d$), and the balance must include that term. If external work *is* done, the system's total energy changes by exactly the energy transferred: $W_{\text{ext}} = \Delta E_{\text{sys}}$. Choosing the system is choosing the bookkeeping – a single object can only *have* kinetic energy; include the Earth or the spring and the system can store potential energy too.



A pendulum trades gravitational potential energy for kinetic energy and back

A potential-energy graph is a complete motion map: the horizontal line at height E is the total energy, the gap $E - U(x)$ is the kinetic energy at each x, and the crossings $E = U$ are the **turning points 转折点** where the object momentarily stops and reverses.

Worked example. A 2.0 kg block slides down a ramp from rest at height 1.5 m, arriving at the bottom at 4.0 m/s. Energy accounting: $mgh = 29.4$ J available; $\frac{1}{2}mv^2 = 16$ J arrives as kinetic energy; so friction converted $29.4 - 16 = 13$ J into thermal energy along the way.

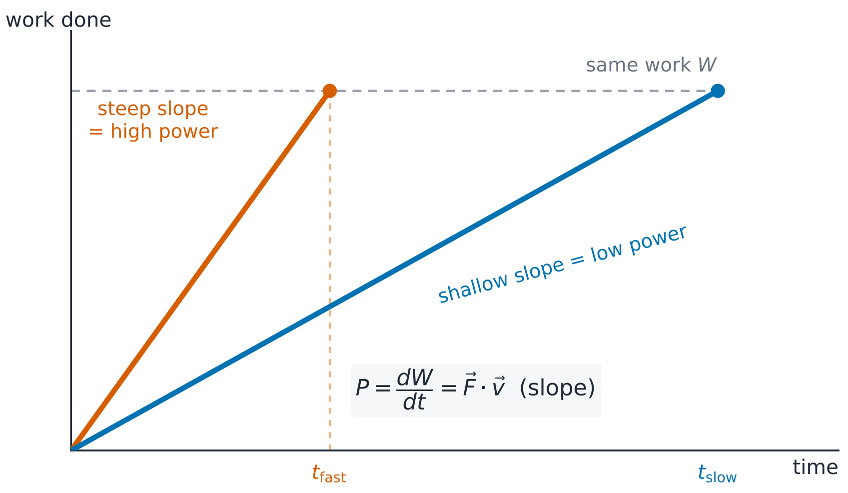
A roller coaster trades energy back and forth: highest (most PE) at the top, fastest (most KE) at the bottom

Power

Power 功率 is the rate of energy transfer, in **watts** 瓦特 (W):

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}, \qquad P_{\text{inst}} = \frac{dW}{dt} = \vec{F}\cdot\vec{v}.$$

It measures how *fast* work is done, not how much. The dot product matters: only the force component along the velocity delivers power.



Power is the slope of the work-time graph: the same work in less time means more power

Worked example. A block released from rest at the top of a frictionless 3.0 m-high ramp reaches the bottom at $v = \sqrt{2gh} = 7.7$ m/s (from $mgh = \frac{1}{2}mv^2$). A motor that then drives it at a steady 7.7 m/s against a 20 N resistance delivers $P = Fv = 20(7.7) \approx 150$ W.

Worked example. A 1200 kg car climbs a hill that rises 1.0 m for every 20 m of road, at a steady 15 m/s. The engine must supply gravity's power drain: $P = mg\,v\sin\theta = 1200(9.8)(15)\left(\frac{1}{20}\right) \approx 8.8$ kW – before adding air resistance.

Exam skill. Energy FRQs reward the *accounting sentence*: name your system, state which forces do work on it, and write the balance ($W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$) before plugging in numbers. "Friction is present, so mechanical energy is not conserved" is a scored statement.

Exam tips

- Use the **work–energy theorem** $W_{net} = \Delta KE$ and compute work as $W = \int \vec{F}\cdot d\vec{r}$ for a variable force.
- Read work off a **force–position graph** as the area under the curve.
- Power is $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$; watch instantaneous vs average.
- Split forces into conservative (define a potential energy) and non-conservative (dissipate energy).
- Choose energy methods over kinematics when the force varies or the path is complex.

Linear Momentum

AP Physics C: Mechanics

Linear Momentum

Linear momentum 动量 is mass times velocity:

$$\vec{p} = m\vec{v}.$$

It is a **vector 矢量** pointing along the velocity, and it is *the* quantity for analysing **collisions 碰撞** and **explosions 爆炸**. A collection of objects can be treated as one system moving with the velocity of its **center of mass 质心**:

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{\sum m_i},$$

so the system's total momentum is its total mass times $\vec{v}_{\text{cm}}$ – one object's worth of bookkeeping for any number of parts.

Change in Momentum and Impulse

Newton's second law is really a statement about momentum:

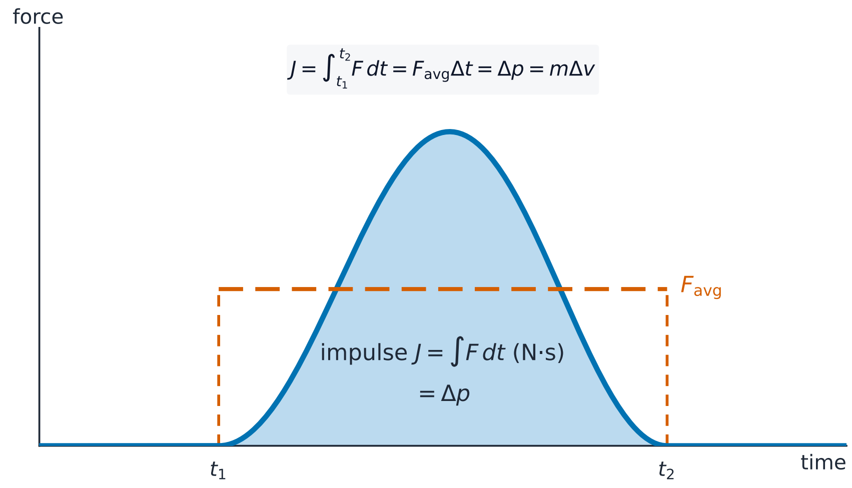
$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt},$$

which reduces to $m\vec{a}$ when the mass is constant – and handles a *changing* mass when it is not (the special case $\vec{F} = \frac{dm}{dt}\vec{v}$ holds only when the velocity is constant, e.g. a chain piling onto a scale or sand landing on a belt moving at fixed speed; a rocket does not fit, since it accelerates). The **impulse 冲量** delivered by a force is its time integral, and the **impulse–momentum theorem 冲量-动量定理** says it equals the change in momentum:

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{\text{net}}\, dt = \Delta\vec{p}.$$

Impulse is a vector along the net force. Read it off graphs both ways:

- On a force–time graph, impulse is the **area** under the curve.
- On a momentum–time graph, the net force is the **slope 斜率** at each instant.



Impulse is the area under the force-time curve, equal to the average force times the contact time

Spreading the same Δp over a longer time lowers the force – that is why airbags, crumple zones, and soft landings work, and why you bend your knees when you land.

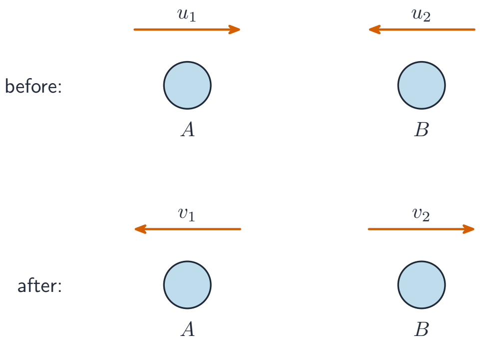
Worked example. A time-varying force $F(t) = 10t$ N acts on a 2.0 kg object for 2.0 s. The impulse is $J = \int_0^2 10t\,dt = \left[5t^2\right]_0^2 = 20$ N · s, so the speed changes by $\Delta v = J/m = 10$ m/s.

Conservation of Linear Momentum

Internal forces come in Newton's-third-law pairs, so they cancel inside any system: they can shuffle momentum *between* parts but never change the total. Momentum only enters or leaves a system through a net *external* force ($\vec{J} = \Delta\vec{p}$). So, with **zero net external force**, total momentum is **conserved 守恒**:

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}.$$

Momentum is conserved in *every* collision, however violent – choose the system large enough that the collision forces are internal. Apply conservation separately along each axis; AP asks for quantitative work in one or two dimensions.



A head-on collision: total momentum before equals total momentum after

Worked example (explosion). A 6.0 kg shell at rest splits into a 2.0 kg piece moving at 9.0 m/s east and a 4.0 kg piece. Total momentum stays zero, so the heavy piece moves west at $v = \dfrac{2.0(9.0)}{4.0} = 4.5$ m/s. The kinetic energy came from stored (chemical or spring) energy – momentum conservation does not require kinetic-energy conservation.

Elastic and Inelastic Collisions

All collisions conserve momentum; they differ in what happens to the **kinetic energy** 动能:

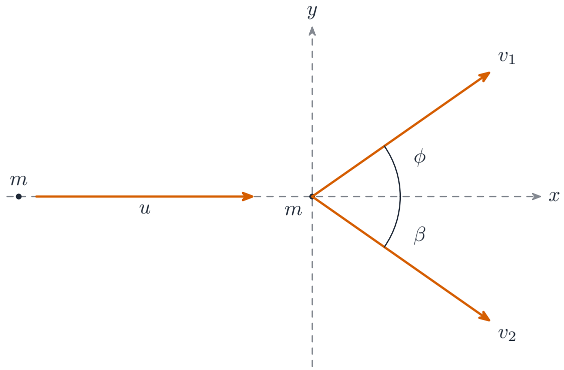
type	momentum	kinetic energy
elastic collision 弹性碰撞	conserved	total conserved (individual shares may change)
inelastic collision 非弹性碰撞	conserved	decreases – some becomes heat, sound, **deformation** 形变
perfectly inelastic collision 完全非弹性碰撞	conserved	largest possible loss – the objects stick and share one velocity

Strategy: *always* write momentum conservation first; add the kinetic-energy equation only when the problem says "elastic". Two useful elastic facts: equal masses in a 1D elastic collision simply exchange velocities, and in the center-of-mass frame each object just reverses its velocity.

Worked example. A 1000 kg car at 20 m/s strikes a stationary 1500 kg car and they lock together – perfectly inelastic. Momentum: $v = \dfrac{1000(20)}{2500} = 8.0$ m/s. Kinetic energy falls from 2.0×10^5 J to $\frac{1}{2}(2500)(8.0)^2 = 8.0 \times 10^4$ J: about 60% is lost, even though momentum is exactly conserved.

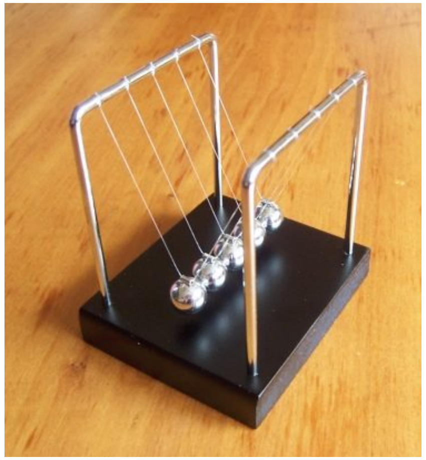
Worked example (2D). A puck moving east at 4.0 m/s strikes an identical puck at rest; after the glancing hit, one moves at 2.0 m/s at 60° north of east. Conserve

each axis: east–west, $m(4.0) = m(2.0)\cos 60° + mv_x$, so $v_x = 3.0$ m/s; north–south, $0 = m(2.0)\sin 60° - mv_y$, so $v_y = 1.7$ m/s. The second puck moves at $\sqrt{3.0^2 + 1.7^2} = 3.5$ m/s, about 30° south of east.



A glancing collision, resolved along two perpendicular axes

Exam skill. On FRQs, justify with the *condition*, not the slogan: "the net external force on the two-puck system is zero during the collision, so its total momentum is constant." If asked whether the collision is elastic, *compute* the kinetic energy before and after and compare – never assume.



A Newton's cradle shows momentum and kinetic energy passing through a near-elastic collision

Exam tips

- Impulse equals the momentum change: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$, and it is the area under a force–time graph.
- **Momentum is conserved** whenever the net external force is zero — always the go-to for collisions.
- Distinguish **elastic** (kinetic energy conserved) from **inelastic** (kinetic energy decreases) collisions; in a **perfectly inelastic** collision the objects stick and move with one velocity.
- Apply conservation to each component (x and y) separately in 2-D.
- Connect to center of mass: total momentum $= M\vec{v}_{cm}$.

Torque and Rotational Dynamics

AP Physics C: Mechanics

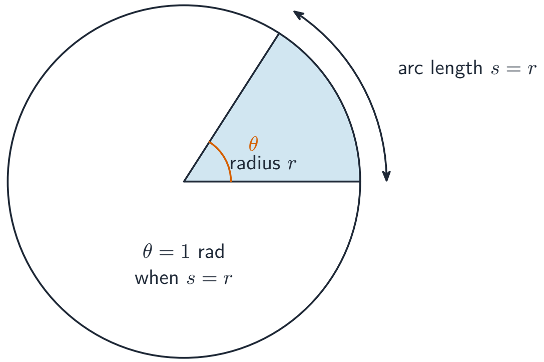
Rotational Kinematics

Rotation mirrors linear motion with angular quantities, defined as derivatives:

$$\omega = \frac{d\theta}{dt}, \qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

Here θ is **angular displacement** 角位移 in **radians** 弧度, ω the **angular velocity** 角速度, and α the **angular acceleration** 角加速度. (AP works with their magnitudes and signs; the 3D vector directions are not assessed.) For constant α, the kinematic equations are the linear ones re-lettered:

$$\omega = \omega_0 + \alpha t, \qquad \Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2, \qquad \omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta.$$



One radian is the angle whose arc length equals the radius

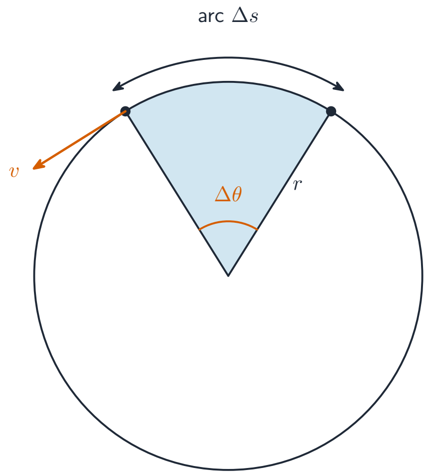
Worked example. A fan blade slows from 30 rad/s to rest with $\alpha = -6.0$ rad/s^2: it takes $t = 5.0$ s and turns through $\Delta\theta = \dfrac{0 - 30^2}{2(-6.0)} = 75$ rad – about 12 revolutions.

Connecting Linear and Rotational Motion

A point at radius r from the axis moves along its arc with

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha,$$

so points farther out move faster. A rotating point generally has *two* acceleration components at once: the **tangential acceleration** 切向加速度 $a_t = r\alpha$ (speeding up along the arc) and the **centripetal acceleration** 向心加速度 $a_c = \dfrac{v^2}{r} = r\omega^2$ (turning, towards the axis). They are perpendicular, so $a = \sqrt{a_t^2 + a_c^2}$.



As the radius turns through an angle, a point moves along an arc at speed v

Torque

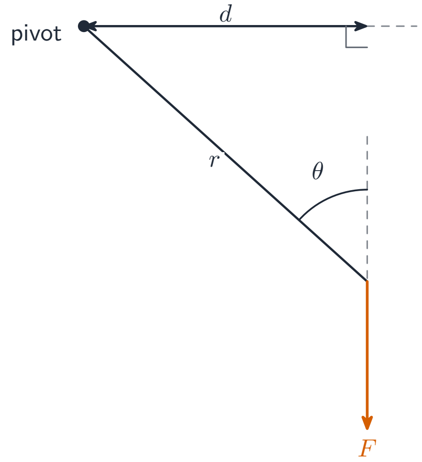
Torque 力矩 is the turning effect of a force – the rotational cause, as force is the translational cause. It is a **vector product** 矢量积:

$$\vec{\tau} = \vec{r} \times \vec{F}, \qquad \tau = rF\sin\theta = F\,r_\perp,$$

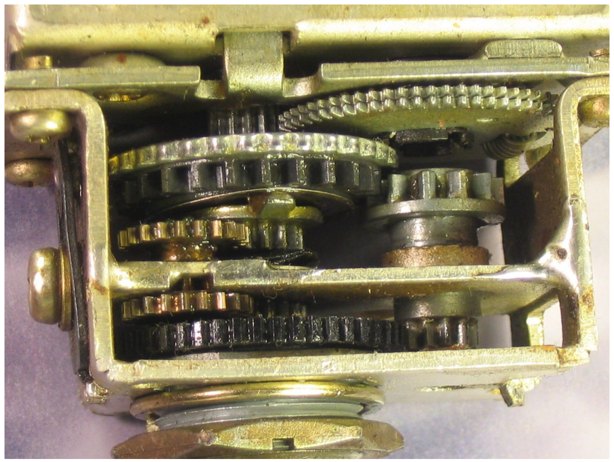
where $r_\perp$, the **moment arm** 力臂, is the perpendicular distance from the axis to the force's line of action. More force, applied farther from the axis, more perpendicular:

more torque. A force pointing straight at (or away from) the axis has zero torque. In a plane, give counterclockwise and clockwise torques opposite signs and add.

As a vector product, $\vec{\tau} = \vec{r} \times \vec{F}$ points **perpendicular to the plane** of $\vec{r}$ and $\vec{F}$, with its direction fixed by the **right-hand rule** 右手定则: curl your right fingers from $\vec{r}$ toward $\vec{F}$ and your thumb points along $\vec{\tau}$ – out of the page for a counterclockwise turn, into it for a clockwise one. The same rule gives the direction of angular momentum $\vec{L} = \vec{r} \times \vec{p}$.



The torque of a force depends on the perpendicular distance from the axis



A gear train: torques and rotational inertia couple through the gear ratio

Rotational Inertia

Rotational inertia 转动惯量 I measures resistance to angular acceleration – the rotational analog of mass, but it depends on *where* the mass sits. For point masses, $I = \sum m_i r_i^2$; for a continuous body,

$$I = \int r^2\, dm.$$

Mass far from the axis dominates, because of the r^2.

Worked example (rod). A uniform rod (mass M, length L) about its centre: with **linear density** 线密度 $\lambda = M/L$, $dm = \lambda\, dx$, so $I = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx = \frac{1}{12} ML^2$. AP also expects *nonuniform* rods: if $\lambda(x) = cx$ on $[0, L]$, first find $M = \int_0^L cx\, dx = \frac{1}{2}cL^2$, then $I = \int_0^L x^2(cx)\, dx = \frac{1}{4}cL^4 = \frac{1}{2}ML^2$ about the light end.

Worked example (disk from rings). A solid disk is a nest of rings: a ring at radius r of width dr has $dm = \frac{M}{\pi R^2} 2\pi r\, dr$, so

$$I = \int_0^R r^2\, dm = \frac{2M}{R^2} \int_0^R r^3\, dr = \tfrac{1}{2} MR^2.$$

The same coaxial-shell method handles cylindrical shells and annular rings.

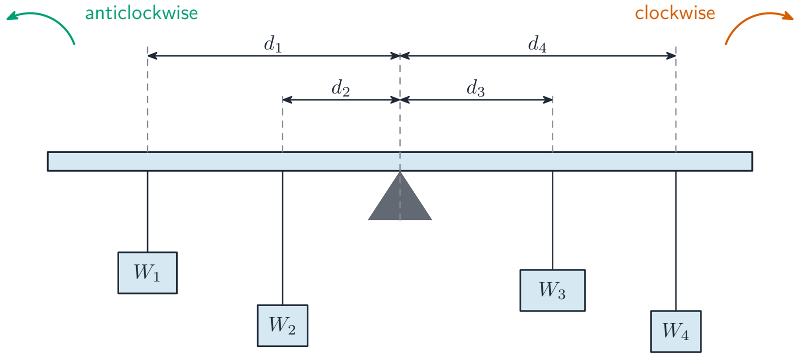
The **parallel-axis theorem** 平行轴定理 shifts any known I to a parallel axis a distance d away:

$$I' = I_{\text{cm}} + Md^2.$$

For the rod about one end: $I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$.

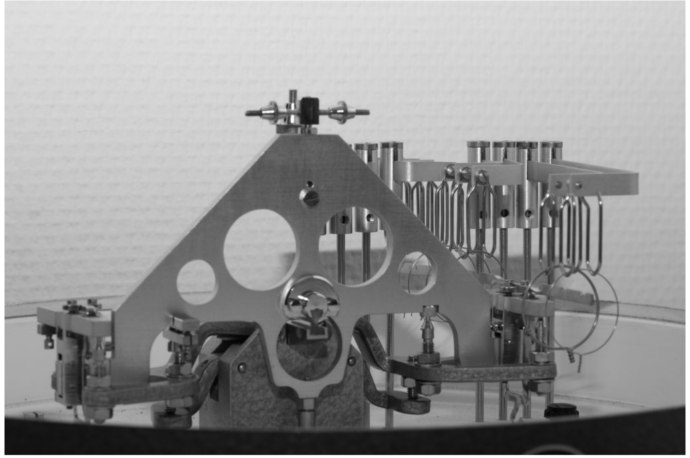
Rotational Equilibrium

Newton's first law has a rotational form: with zero net torque, angular velocity is constant – an object in **rotational equilibrium** 转动平衡 either does not rotate or rotates steadily. Full **static equilibrium** 静平衡 needs both $\sum F = 0$ and $\sum \tau = 0$ (about *any* axis). The standard move for beams and ladders: put the axis at the point where an unknown force acts, so that force drops out of the torque equation.



At balance the clockwise and anticlockwise torques about the axis are equal

Worked example. A uniform 20 kg beam, 4.0 m long, is pivoted at its left end and held horizontal by a vertical rope at its right end. A 40 kg child sits 1.0 m from the pivot. Torques about the pivot: $T(4.0) = 20g(2.0) + 40g(1.0)$, so $T = \frac{392 + 392}{4.0} = 196$ N. Then vertical force balance gives the pivot's upward push: $F_p = (60)(9.8) - 196 = 392$ N. Choosing the pivot as the axis removed F_p from the torque equation entirely.



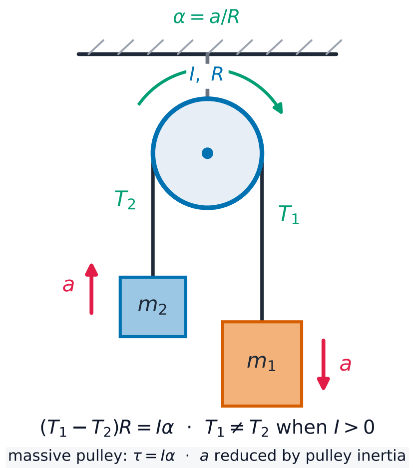
A beam balance: rotational equilibrium when clockwise and anticlockwise torques match

Newton's Second Law in Rotational Form

A net torque produces angular acceleration in proportion to the rotational inertia:

$$\alpha = \frac{\sum \tau}{I} \qquad \left(\text{more generally } \sum \vec{\tau} = \frac{d\vec{L}}{dt}\right).$$

Solve rotational dynamics exactly like translational dynamics, with $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$ – free-body diagram, equations, solve.



A massive pulley: the two tensions differ because the pulley needs net torque

Worked example (massive pulley). Masses $m_1 = 4.0$ kg and $m_2 = 2.0$ kg hang from a rope over a **pulley** 滑轮 of rotational inertia $I = 0.50\ \text{kg}\cdot\text{m}^2$ and radius $R = 0.20$ m. Three Newton's-law equations, one per body:

$$m_1 g - T_1 = m_1 a, \qquad T_2 - m_2 g = m_2 a, \qquad (T_1 - T_2)R = I\alpha = \frac{Ia}{R}.$$

Adding them (with $I/R^2 = 12.5$ kg): $a = \dfrac{(m_1 - m_2)g}{m_1 + m_2 + I/R^2} = \dfrac{19.6}{18.5} = 1.1\ \text{m/s}^2$. Then $T_1 = m_1(g - a) = 35$ N and $T_2 = m_2(g + a) = 22$ N. The tensions *must* differ – otherwise nothing would spin the pulley. (If a problem says the pulley is light, then $I \approx 0$ and the tensions become equal again.)

Exam skill. Rotational FRQs score the setup: separate free-body diagrams for each mass *and* the pulley, the string constraint $a = R\alpha$ stated, and the sign convention

consistent. The single most common error is assuming one tension throughout a rope that passes over a *massive* pulley.

Exam tips

- Use the rotational analogues: $\theta, \omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt}$, with the constant-$\alpha$ equations mirroring linear kinematics.
- Convert between linear and angular with $v = r\omega$ and $a_t = r\alpha$ (plus centripetal $a_c = \frac{v^2}{r} = \omega^2 r$).
- Keep radians throughout and fix a positive sense of rotation.
- Relate torque to angular acceleration through $\tau = I\alpha$.
- Draw the rotation axis — the moment arm is the perpendicular distance to it.

Energy and Momentum of Rotating Systems

AP Physics C: Mechanics

Rotational Kinetic Energy

A spinning body has **rotational kinetic energy** 转动动能

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2,$$

the rotational twin of $\frac{1}{2}mv^2$, with the **rotational inertia** 转动惯量 I playing the role of mass. A body that both moves and spins carries **both** terms:

$$K = \tfrac{1}{2}mv_{\text{cm}}^2 + \tfrac{1}{2}I\omega^2.$$

Worked example. A uniform cylinder ($I = \frac{1}{2}mr^2$) rolls at speed v. Its kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2$ – one third of it is rotational. The same ball of energy bookkeeping decides every rolling problem.

Torque and Work

A **torque** 力矩 acting through an **angular displacement** 角位移 does work, and power is torque times angular velocity:

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta, \qquad P = \tau\omega.$$

These are the rotational forms of $W = \int F\,dx$ and $P = Fv$ – the whole translational energy toolkit carries over with $F \to \tau$, $x \to \theta$, $v \to \omega$.

Worked example. A motor applies a constant $8.0\ \text{N}\cdot\text{m}$ torque to a flywheel for 5.0 full turns: $W = \tau\,\Delta\theta = 8.0(5.0)(2\pi) = 250$ J, which appears as rotational kinetic energy if friction is negligible.

Angular Momentum and Angular Impulse

Angular momentum 角动量 for a particle is

$$\vec{L} = \vec{r} \times \vec{p}, \qquad |L| = mvr\sin\theta,$$

so even a particle moving in a straight line has angular momentum about any point not on that line ($L = mv\,d$, with d the perpendicular distance). For a rigid body spinning about a fixed axis, $L = I\omega$. Newton's second law in rotational form is

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} \quad (= I\alpha \text{ when } I \text{ is constant}),$$

and a net torque acting over time delivers an **angular impulse** 角冲量 $\int \tau\,dt = \Delta L$ – the rotational impulse–momentum theorem.

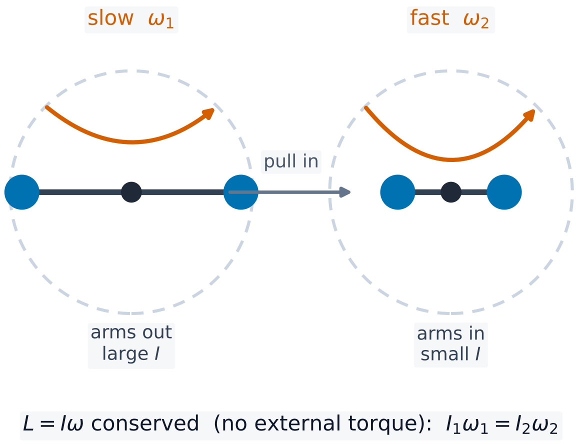
Conservation of Angular Momentum

With **zero net external torque**, total angular momentum is **conserved** 守恒:

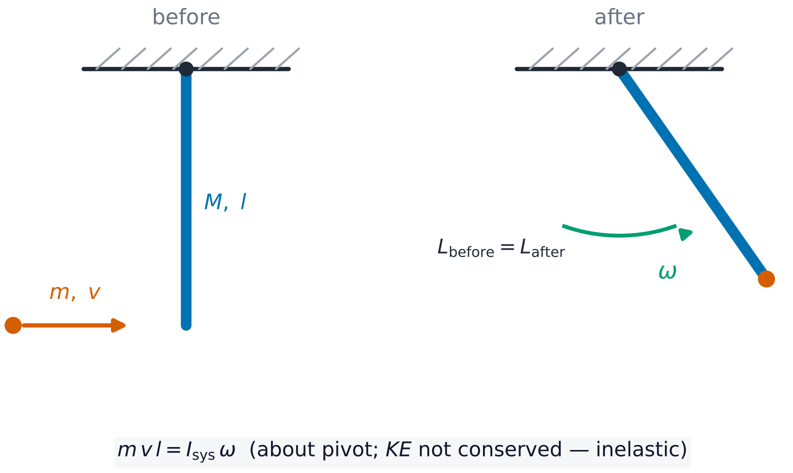
$$L_{\text{before}} = L_{\text{after}}, \qquad I_1\omega_1 = I_2\omega_2 \text{ (one rigid body reshaping).}$$

If I shrinks, ω grows – a spinning skater speeds up pulling her arms in. The rule survives collisions and shape changes, which is what makes it so useful: pick the axis so that every external force (gravity, the pivot force) exerts no torque about it.

Worked example. A skater spins at 2.0 rev/s with $I_1 = 4.0\ \text{kg}\cdot\text{m}^2$. Pulling in her arms drops it to $I_2 = 1.6\ \text{kg}\cdot\text{m}^2$: $\omega_2 = \frac{4.0}{1.6}(2.0) = 5.0$ rev/s. Her kinetic energy *rises* – the extra energy is the work her muscles do pulling her arms inward.



Pulling mass inward lowers I, so ω rises to conserve $L = I\omega$



Angular momentum about the pivot is conserved as the bullet embeds in the rod

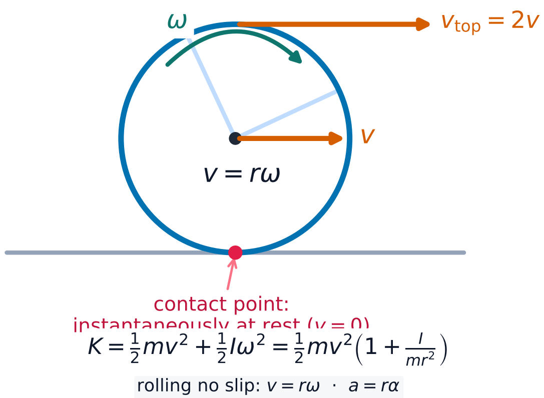
Worked example (rotational collision). A 0.020 kg bullet at 300 m/s strikes the tip of a uniform rod ($M = 1.5$ kg, length $l = 0.60$ m) hanging from a pivot, and embeds. About the pivot, gravity and the pivot force exert no torque during the strike, so L is conserved: $L = mvl = 0.020(300)(0.60) = 3.6\ \text{kg}\cdot\text{m}^2/\text{s}$. Afterwards $I = \frac{1}{3}Ml^2 + ml^2 = 0.18 + 0.0072 = 0.187\ \text{kg}\cdot\text{m}^2$, so $\omega = \frac{3.6}{0.187} \approx 19$ rad/s. (Linear momentum is *not* conserved here – the pivot pushes on the rod – and kinetic energy certainly is not: check both before claiming them.)

Figure skating and angular momentum: a skater who pulls their arms in during a spin lowers their rotational inertia, so they spin faster

Rolling

Rolling without slipping 无滑滚动 ties translation to rotation: the contact point is momentarily at rest, so

$$\Delta x_{\text{cm}} = r\,\Delta\theta, \qquad v_{\text{cm}} = r\omega, \qquad a_{\text{cm}} = r\alpha.$$



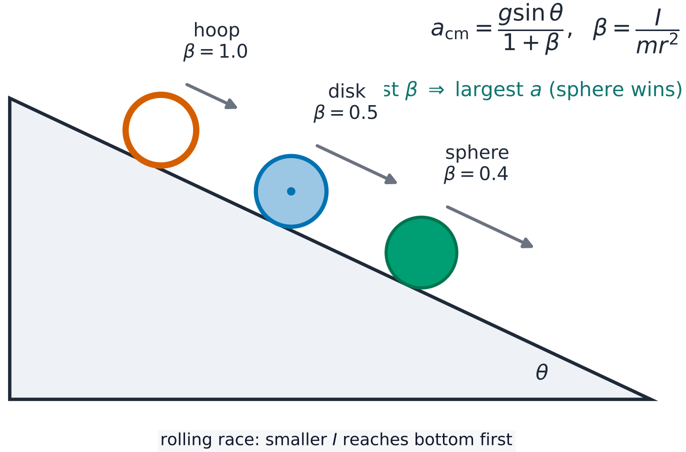
In rolling without slipping the contact point is at rest, so $v = r\omega$

Static friction supplies the torque that keeps the spin matched to the motion, but at a point that is not sliding – so for pure rolling, friction does **no work**, and energy conservation is safe to use. (Rolling friction is beyond the AP course.)

Racing shapes down an incline shows the energy split. With $I = \beta mr^2$, energy conservation gives

$$a_{\text{cm}} = \frac{g\sin\theta}{1+\beta}:$$

a **sphere** ($\beta = \frac{2}{5}$) beats a **disk** 圆盘 ($\beta = \frac{1}{2}$), which beats a **hoop** 圆环 ($\beta = 1$) – mass and radius cancel completely. More of the hoop's energy is locked in rotation, so its center moves slower.



Rolling race: the shape with the smallest I/mr^2 reaches the bottom first

Worked example. A solid sphere rolls from rest down a 30° incline: $a = \dfrac{g\sin 30^\circ}{1+\frac{2}{5}} = \dfrac{9.8(0.50)}{1.4} = 3.5\ \text{m/s}^2$, versus $4.9\ \text{m/s}^2$ for a frictionless slider – rolling objects always lose the race against sliding ones.

Motion of Orbiting Satellites

Gravity supplies the **centripetal force** 向心力 for a **satellite** 卫星 in a circular orbit:

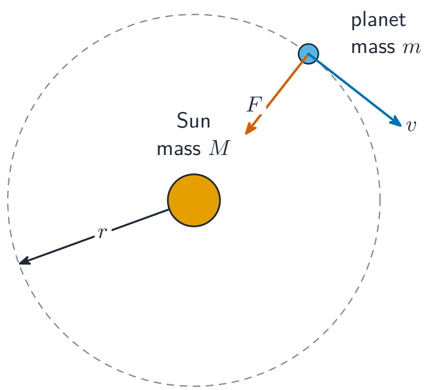
$$\frac{GMm}{r^2} = \frac{mv^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

– larger orbits are slower. With **gravitational potential energy** 引力势能 $U_g = -\frac{GMm}{r}$, a circular orbit obeys $K = -\frac{1}{2}U$, so the **total mechanical energy** 总机械能 is

$$E = K + U = \frac{U}{2} = -\frac{GMm}{2r},$$

negative because the satellite is bound. To escape from radius r, total energy must reach zero, giving the **escape velocity** 逃逸速度

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}.$$



Gravity provides the centripetal force that keeps a satellite in orbit

In an **elliptical orbit** 椭圆轨道, E and L are fixed: gravity points at the focus, so it exerts no torque about it. Conserved L means the satellite sweeps equal areas in equal times – **Kepler's second law** 开普勒第二定律 – and therefore moves fastest at closest approach, slowest at the far point. Energy conservation connects speeds at the two ends.

Worked example. For a satellite at $r = 7.0 \times 10^6$ m around Earth ($GM = 4.0 \times 10^{14}$ m^3/s^2): orbital speed $v = \sqrt{GM/r} = 7.6 \times 10^3$ m/s, while escaping from that radius needs $v_{\text{esc}} = \sqrt{2GM/r} = 1.1 \times 10^4$ m/s – exactly $\sqrt{2}$ times the circular speed.

Exam skill. Orbit FRQs are energy-and-angular-momentum problems in disguise: write $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$ and $L = mvr\sin\theta$ at the two points of interest and solve the pair – never assume the orbit formulae for circles apply to an ellipse.

Exam tips

- Compute the **moment of inertia** $I = \int r^2\,dm$ and shift axes with the **parallel-axis theorem** $I = I_{cm} + Md^2$.
- Rotational kinetic energy is $\frac{1}{2}I\omega^2$; a rolling body has both translational and rotational KE.
- Conserve **angular momentum** $L = I\omega$ when net external torque is zero (a spinning skater pulling in).
- Use energy conservation for rolling-without-slipping problems ($v = r\omega$ ties the two motions).
- Know standard I values (hoop, disk, rod, sphere) and where the axis is.

Oscillations

AP Physics C: Mechanics

Defining Simple Harmonic Motion

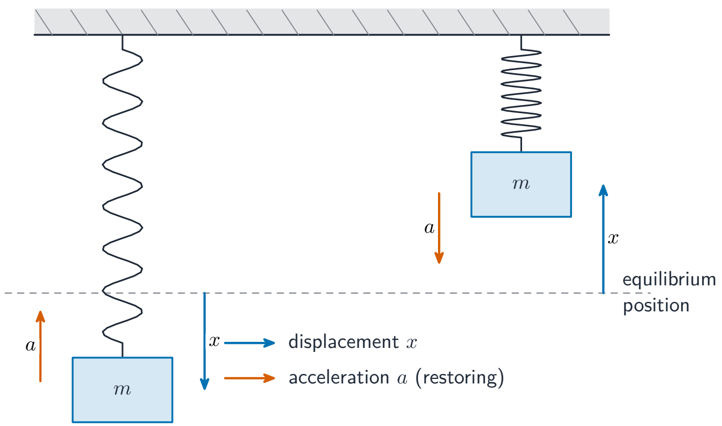
Simple harmonic motion 简谐运动 (SHM) is a special case of **periodic motion** 周期运动. It appears whenever two things are true: there is an **equilibrium** 平衡位置 position where the net force is zero, and displacing the object produces a **restoring force** 回复力 – a force pointing back toward equilibrium – whose magnitude is *proportional* to the displacement:

$$F = -k\,\Delta x.$$

Newton's second law then gives the defining **differential equation** 微分方程:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x, \qquad \omega = \sqrt{\frac{k}{m}}.$$

Its solution is sinusoidal, $x(t) = A\cos(\omega t + \phi)$. You do not need to prove this solution – you need to *recognise* the equation: any system whose motion obeys "$\ddot{x} = -\omega^2 x$" is a simple harmonic oscillator, whatever it is made of. (A mass hanging on a vertical spring works the same way: gravity only shifts the equilibrium point; the oscillation about it is unchanged.)



In SHM the acceleration always points back towards equilibrium, opposite the displacement

Frequency and Period of SHM

The **angular frequency** 角频率 ω sets the **period** 周期 and **frequency** 频率:

$$T = \frac{2\pi}{\omega} = \frac{1}{f}.$$

For the two standard systems:

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}, \qquad T_{\text{pendulum}} = 2\pi\sqrt{\frac{\ell}{g}}.$$

Two classic conceptual traps: the period of SHM never depends on the **amplitude** 振幅, and each system ignores one obvious variable – the pendulum's period does not involve its mass, and the spring's period does not involve g (a spring–block oscillator keeps perfect time in orbit; a pendulum clock does not).

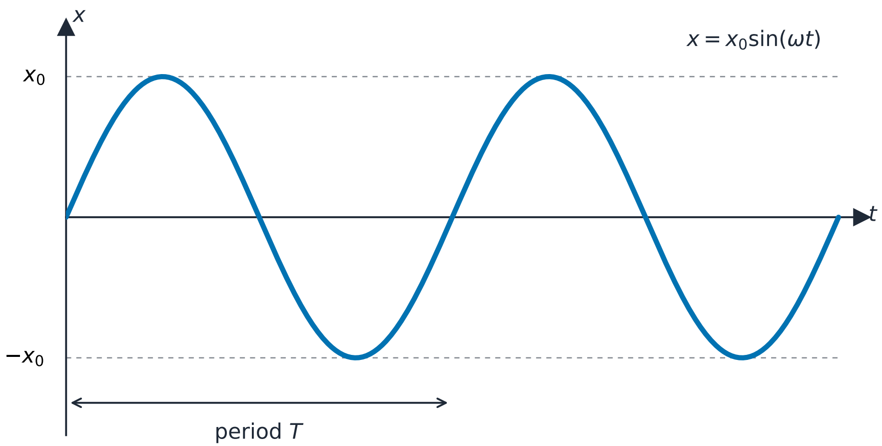
Representing and Analyzing SHM

Differentiate $x(t) = A\cos(\omega t + \phi)$ twice:

$$v = -A\omega\sin(\omega t + \phi), \qquad a = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x.$$

So at the ends of the swing ($x = \pm A$) the speed is zero and the acceleration is largest; passing through equilibrium ($x = 0$) the acceleration is zero and the speed is largest, $v_{\max} = A\omega$. The amplitude A and the **phase constant** 相位常数 ϕ come from the initial conditions: where the object starts and how fast it is moving. Eliminating t (or using energy, below) gives speed as a function of position:

$$v = \pm\omega\sqrt{A^2 - x^2}.$$



Displacement varies sinusoidally with time in simple harmonic motion

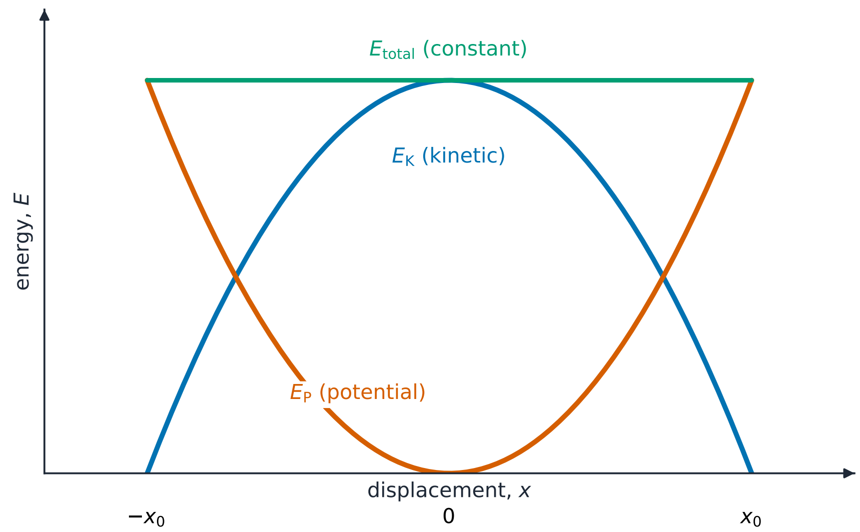
Worked example. A 0.50 kg mass on a $k = 200$ N/m spring: $\omega = \sqrt{k/m} = 20$ rad/s, $T = 2\pi/\omega = 0.31$ s. With $A = 0.10$ m: $v_{\max} = A\omega = 2.0$ m/s at equilibrium, and at $x = 0.050$ m the speed is $v = \omega\sqrt{A^2 - x^2} = 20\sqrt{0.10^2 - 0.050^2} = 1.7$ m/s.

Energy of Simple Harmonic Oscillators

The total energy of the oscillator is the sum $E = U + K$, and with no friction it is constant – energy just trades back and forth between the spring's potential energy and the mass's kinetic energy:

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2.$$

All potential at the ends (where $K = 0$), all kinetic at equilibrium (where U is minimum). Because $E \propto A^2$, doubling the amplitude quadruples the energy.



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example. Where is the energy split evenly? Set $\frac{1}{2}kx^2 = \frac{1}{2}E = \frac{1}{4}kA^2$, so $x = A/\sqrt{2} \approx 0.71A$ – much closer to the end than to the middle. For the system above ($A = 0.10$ m): $x = 0.071$ m.

Left alone, a system oscillates at its own **natural frequency** 固有频率 (set by k and m: $f_0 = \frac{1}{2\pi}\sqrt{k/m}$). Drive it with a periodic force at that same frequency and the amplitude grows dramatically – **resonance** 共振. Each well-timed push adds energy faster than damping removes it, so the driven amplitude peaks sharply when the driving frequency matches f_0 (a swing pumped in time, a bridge shaken by wind).

Simple and Physical Pendulums

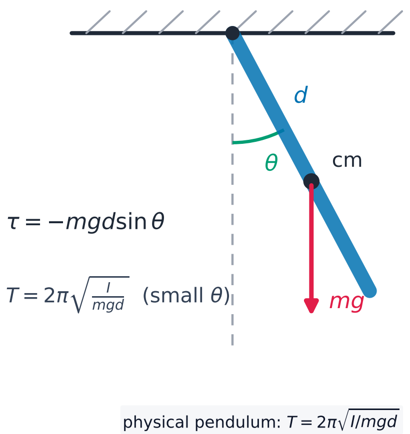
A **physical pendulum** 物理摆 is any rigid body swinging about a fixed pivot. Displace it by an angle θ and gravity, acting at the center of mass a distance d from the pivot, supplies a restoring torque

$$\tau = -mgd\sin\theta.$$

For small angles, apply the **small-angle approximation** 小角度近似 $\sin\theta \approx \theta$ and Newton's second law in rotational form ($\tau = I\alpha$):

$$\frac{d^2\theta}{dt^2} = -\frac{mgd}{I}\theta = -\omega^2\theta \quad \Rightarrow \quad T_{\text{phys}} = 2\pi\sqrt{\frac{I}{mgd}}.$$

This is the same "$\ddot{\theta} = -\omega^2\theta$" pattern as before – recognising it *is* the derivation. A **simple pendulum** 单摆 is the special case of a point mass on a light string: $I = m\ell^2$ and $d = \ell$ give $T = 2\pi\sqrt{\ell/g}$.



Gravity acting at the center of mass provides a physical pendulum's restoring torque

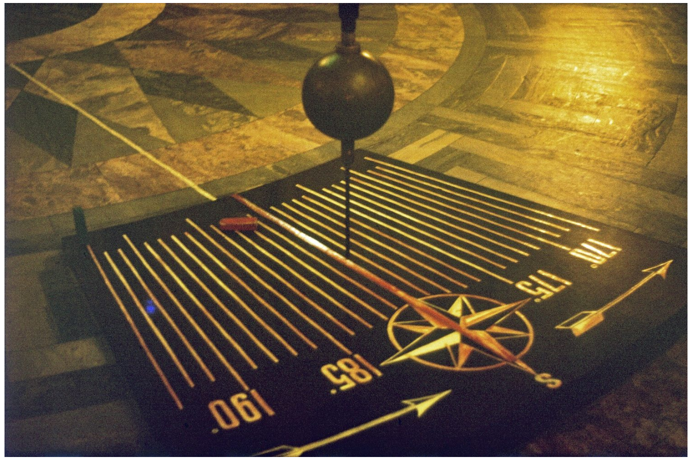
Worked example. A uniform rod (mass M, length L) swings from one end: $I = \frac{1}{3}ML^2$, $d = \frac{L}{2}$, so

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\,\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

– shorter than a simple pendulum of length L, because the rod's mass sits nearer the pivot.

A **torsion pendulum** 扭摆 – a disk hanging from a wire that twists – is SHM one more time: the wire's restoring torque is proportional to the twist angle, $I\alpha = -\kappa\,\Delta\theta$, giving $T = 2\pi\sqrt{I/\kappa}$.

Exam skill. Every pendulum FRQ wants the same three steps: write the restoring torque about the pivot, apply the small-angle approximation, and match the result to $\ddot{\theta} = -\omega^2\theta$ to read off ω. State the small-angle step explicitly – it is a scored point, and it is why large-amplitude swings are *not* simple harmonic.



A Foucault pendulum: a long, heavy pendulum whose slow swing reveals the Earth turning beneath it

Exam tips

- Identify SHM from a **linear restoring force** $F = -kx$, which gives $\frac{d^2x}{dt^2} = -\omega^2 x$ with $\omega = \sqrt{k/m}$.
- Write the solution $x = A\cos(\omega t + \phi)$ and get v, a by differentiating; period $T = \frac{2\pi}{\omega}$ is amplitude-independent.
- Energy trades between $\frac{1}{2}kx^2$ and $\frac{1}{2}mv^2$, with total $\frac{1}{2}kA^2$.
- For a pendulum, use the small-angle approximation $\sin\theta \approx \theta$ to reach SHM.
- Match phase to the start: released from rest at maximum displacement uses cosine.