Work, Energy, and Power

AP Physics C: Mechanics

Translational Kinetic Energy

Kinetic energy 动能 is the energy of motion, a **scalar** 标量 measured in **joules** 焦耳 (J):

$$K = \tfrac{1}{2}mv^2.$$

It grows with the *square* of speed –doubling the speed quadruples K. One subtlety worth knowing: kinetic energy depends on the observer's **reference frame** 参考系. A passenger walking down a train has a small K measured inside the train and a huge one measured from the ground –both observers are right, each in their own frame.

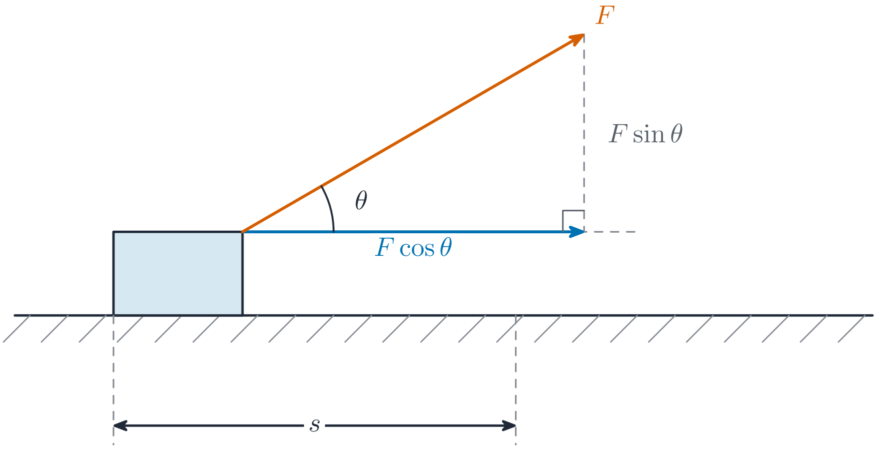
Work

Work 功 is energy transferred into or out of a system by a force acting over a distance. For a variable force it is an **integral** 积分 along the path:

$$W = \int_a^b \vec{F} \cdot d\vec{r} = \int F \cos\theta \, dr.$$

For a constant force this reduces to $W = Fd\cos\theta$; on a force–position graph, work is the **area** under the curve. Work is a scalar with a sign, and the sign is physics, not bookkeeping:

- **Positive** –force has a component along the motion (it speeds the object up).
- **Negative** –force opposes the motion (**friction** 摩擦力 and air drag do negative work).
- **Zero** –force perpendicular to the motion (the **normal force** 法向力 on a sliding block, gravity on a horizontal move, tension in a circular swing).



Only the force component along the displacement does work

The **work–energy theorem** 动能定理 collects every force's contribution: the *net* work equals the change in kinetic energy,

$$W_{\text{net}} = \sum W_i = \Delta K.$$

Worked example. A variable force $F(x) = 3x^2$ N (along the motion) acts from $x = 0$ to $x = 2$ m: $W = \displaystyle\int_0^2 3x^2\,dx = \left[x^3\right]_0^2 = 8$ J. Acting alone on a body starting from rest, it would raise the kinetic energy to exactly 8 J.

Potential Energy

Potential energy 势能 is energy a *system* stores by the positions of its parts –it exists only for **conservative forces** 保守力, whose work is path independent. It is defined through work:

$$\Delta U = -\int_a^b \vec{F} \cdot d\vec{r},$$

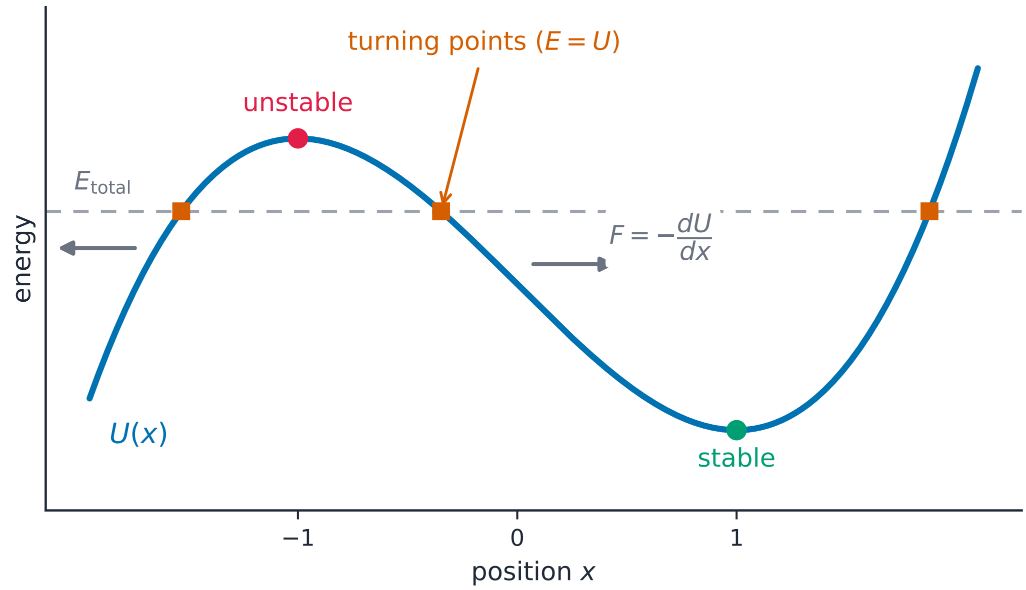
and you are free to *choose where* $U = 0$ –only changes in U matter, so pick the zero that makes the problem simplest. The standard results:

- Gravity near a surface: $\Delta U_g = mg\,\Delta y$.
- Gravity in general: $U_g = -\dfrac{Gm_1m_2}{r}$ (zero at infinite separation).
- Spring: $U_s = \frac{1}{2}k(\Delta x)^2$, with Δx measured from natural length.

For systems of several objects, add the potential energy of each *pair*. Turning the definition around, a conservative force is minus the **derivative** 导数 of its potential energy:

$$F_x = -\frac{dU}{dx}.$$

The force points "downhill" on the $U(x)$ curve. Equilibrium sits where the slope is zero: a minimum is a **stable equilibrium** 稳定平衡 (displaced, the force pushes back), a maximum is an **unstable equilibrium** 不稳定平衡 (displaced, the force pushes away).



On a potential-energy curve the force points downhill and $E = U$ marks the turning points

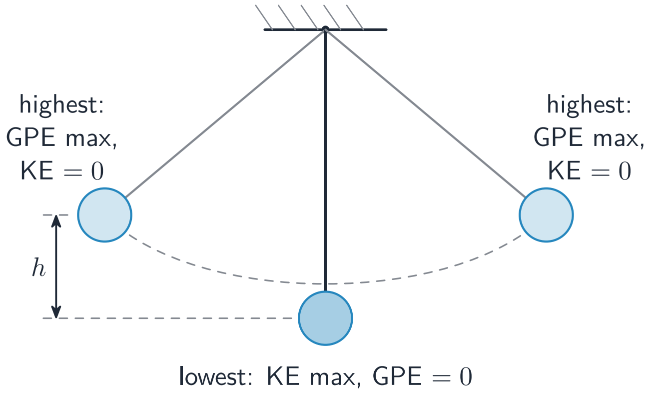
Worked example. Given $U(x) = 2x^3 - 6x$ (joules), the force is $F = -\dfrac{dU}{dx} = 6 - 6x^2$. Equilibria sit at $F = 0$: $x = \pm 1$. Since $\dfrac{d^2U}{dx^2} = 12x$ is positive at $x = +1$ (a minimum –stable) and negative at $x = -1$ (a maximum –unstable), the two points behave oppositely.

Conservation of Energy

Energy is conserved in *all* interactions –the question is only where it goes. **Mechanical energy** 机械能 is the sum $E = K + U$. If the external work on a system is zero and nothing inside it acts through **nonconservative forces** 非保守力, then

$$K_1 + U_1 = K_2 + U_2.$$

When friction or drag act, they convert mechanical energy into **thermal energy** 热能 ($\Delta E_{\text{mech}} = -F_f d$), and the balance must include that term. If external work *is* done, the system's total energy changes by exactly the energy transferred: $W_{\text{ext}} = \Delta E_{\text{sys}}$. Choosing the system is choosing the bookkeeping –a single object can only *have* kinetic energy; include the Earth or the spring and the system can store potential energy too.



A pendulum trades gravitational potential energy for kinetic energy and back

A potential-energy graph is a complete motion map: the horizontal line at height E is the total energy, the gap $E - U(x)$ is the kinetic energy at each x, and the crossings $E = U$ are the **turning points** 转折点 where the object momentarily stops and reverses.

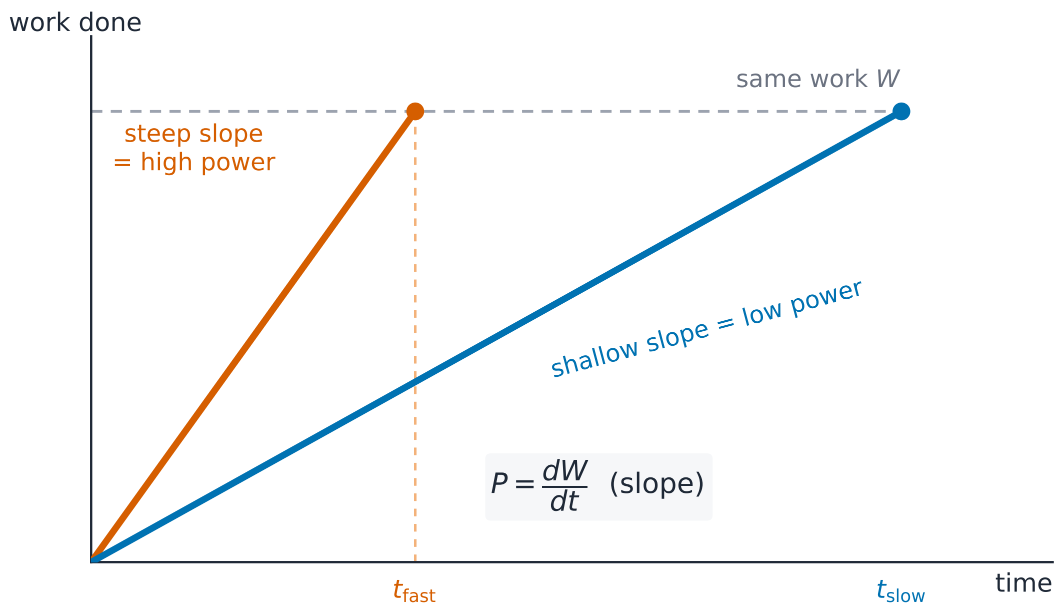
Worked example. A 2.0 kg block slides down a ramp from rest at height 1.5 m, arriving at the bottom at 4.0 m/s. Energy accounting: $mgh = 29.4$ J available; $\frac{1}{2}mv^2 = 16$ J arrives as kinetic energy; so friction converted $29.4 - 16 = 13$ J into thermal energy along the way.

Power

Power 功率 is the rate of energy transfer, in **watts** 瓦特 (W):

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t}, \qquad P_{\text{inst}} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}.$$

It measures how *fast* work is done, not how much. The dot product matters: only the force component along the velocity delivers power.



Power is the slope of the work-time graph: the same work in less time means more power

Worked example. A block released from rest at the top of a frictionless 3.0 m-high ramp reaches the bottom at $v = \sqrt{2gh} = 7.7$ m/s (from $mgh = \frac{1}{2}mv^2$). A motor that then drives it at a steady 7.7 m/s against a 20 N resistance delivers $P = Fv = 20(7.7) \approx 150$ W.

Worked example. A 1200 kg car climbs a hill that rises 1.0 m for every 20 m of road, at a steady 15 m/s. The engine must supply gravity's power drain: $P = mg\,v\sin\theta = 1200(9.8)(15)\left(\frac{1}{20}\right) \approx 8.8$ kW –before adding air resistance.

Exam skill. Energy FRQs reward the *accounting sentence*: name your system, state which forces do work on it, and write the balance ($W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$) before plugging in numbers. "Friction is present, so mechanical energy is not conserved" is a scored statement.

Exam tips

- Use the **work–energy theorem** $W_{net} = \Delta KE$ and compute work as $W = \int \vec{F} \cdot d\vec{r}$ for a variable force.
- Read work off a **force–position graph** as the area under the curve.
- Power is $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$; watch instantaneous vs average.
- Split forces into conservative (define a potential energy) and non-conservative (dissipate energy).
- Choose energy methods over kinematics when the force varies or the path is complex.