Force and Translational Dynamics

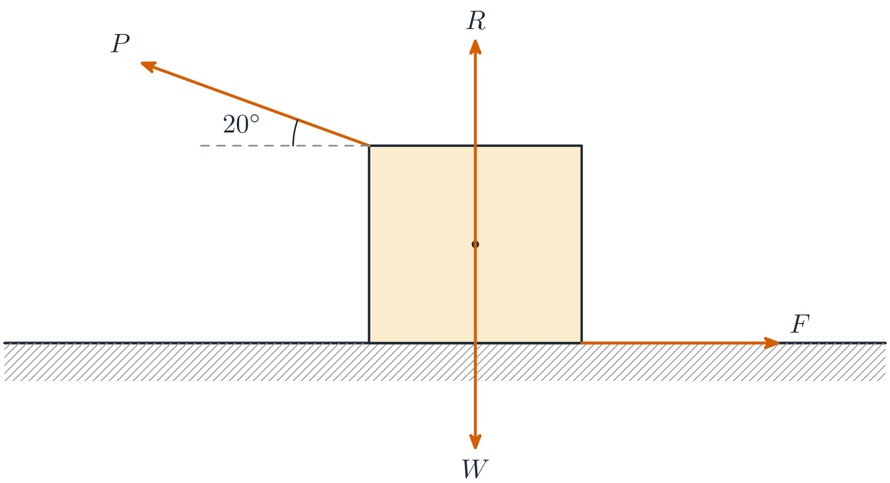
AP Physics C: Mechanics

Systems and Center of Mass

A **system** 系统 is the object or objects you analyze, treated as a point at its **center of mass** 质心. For a set of masses, $x_{\text{cm}} = \dfrac{\sum m_i x_i}{\sum m_i}$; for a continuous body, $x_{\text{cm}} = \dfrac{1}{M}\int x\,dm$. Only **external** forces move the center of mass.

Forces and Free-Body Diagrams

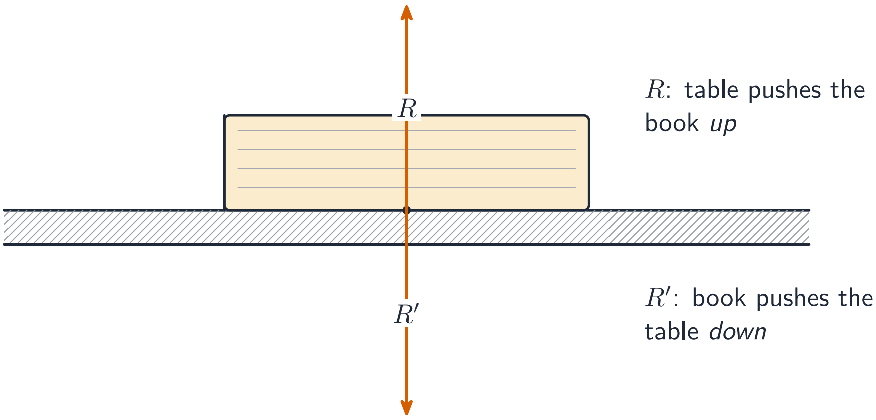
A **force** 力 is a push or pull (a vector, in newtons). A **free-body diagram** 受力图 draws one object with an arrow for **every** force on it –weight, normal, tension, friction, applied, drag. Draw it first; it sets up every dynamics equation.



A free-body diagram shows every force acting on one object

Newton's Third Law

Newton's third law 牛顿第三定律: A pushes B, B pushes back equally and oppositely. The pair acts on **different** objects, so it never cancels within one free-body diagram.



A Newton's third-law pair: equal and opposite forces on two different objects

Newton's First Law

Newton's first law (**inertia** 惯性): with zero **net force** 合力, velocity stays constant –the object is in **translational equilibrium** 平动平衡.

Newton's Second Law

The general form uses momentum:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \quad \text{(for constant mass)}.$$

Apply it one axis at a time. When forces depend on velocity or position, this becomes a **differential equation** 微分方程 to solve.

Worked example. A 2.0 kg block slides down a frictionless **incline** 斜面 at 30°. Only the along-ramp component of gravity drives it, so $a = g \sin 30^\circ = 9.8(0.5) = 4.9\ \mathrm{m/s^2}$, independent of the mass.

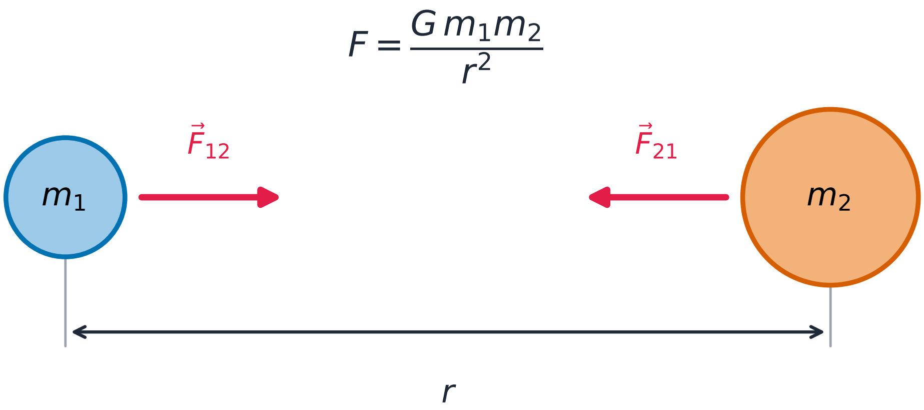
Worked example (two bodies). A 6.0 kg cart on a frictionless table is pulled by a string over a light pulley to a hanging 2.0 kg mass. Treat the pair as one system: only the hanging weight drives it, so $a = \dfrac{2.0(9.8)}{8.0} = 2.45\ \mathrm{m/s^2}$. Then isolate the cart alone to find the string **tension** 张力: $T = 6.0(2.45) \approx 15$ N. System first for a, single body for internal forces –that two-step is the standard pattern.

Gravitational Force

Near a surface, weight is $F_g = mg$. In general, **Newton's law of gravitation** 万有引力定律:

$$F_g = \frac{Gm_1m_2}{r^2},$$

attractive and inverse-square. The gravitational field is $g = \dfrac{GM}{r^2}$.



Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

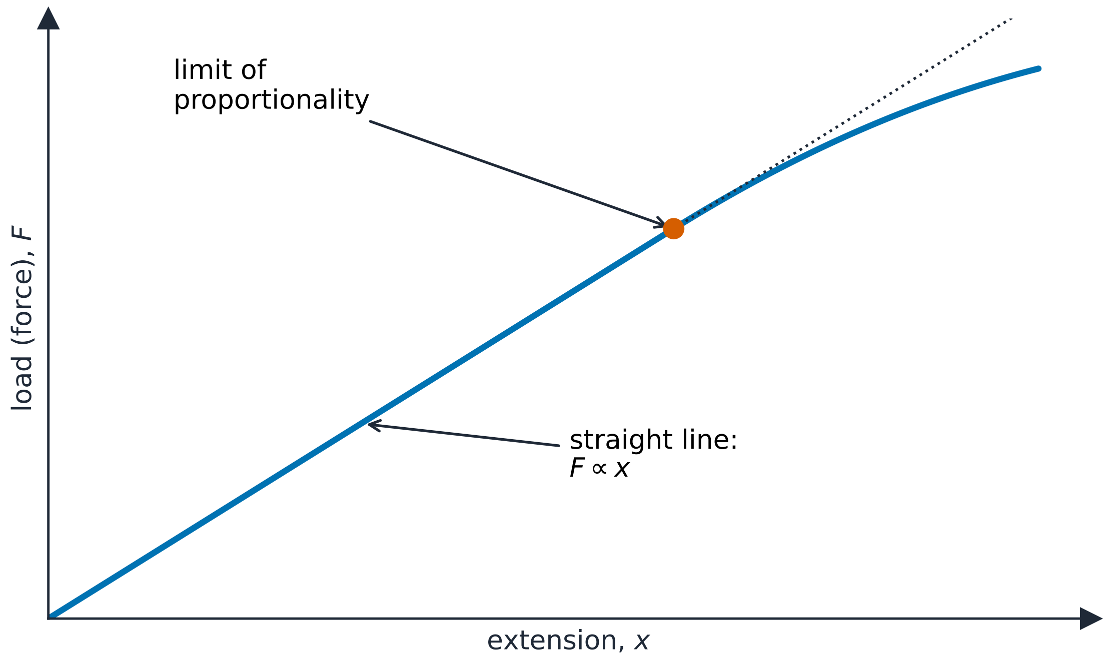
Kinetic and Static Friction

Friction 摩擦力 opposes sliding along a surface: **kinetic** 动摩擦 $f_k = \mu_k N$ while sliding, and **static** 静摩擦 $f_s \le \mu_s N$ up to a maximum before sliding. N is the **normal force** 法向力. Note the inequality: static friction is only as large as it needs to be, and $\mu_s N$ is its *ceiling*, not its value.

Worked example. The same block on the same 30° incline, now with $\mu_k = 0.20$. Perpendicular to the ramp: $N = mg\cos 30°$. Along the ramp: $ma = mg\sin 30° - \mu_k mg\cos 30°$, so $a = g(\sin 30° - 0.20\cos 30°) = 9.8(0.500 - 0.173) = 3.2\ \mathrm{m/s^2}$ –the mass cancels again.

Spring Forces

An ideal spring obeys **Hooke's law** 胡克定律 $F_s = -kx$, a restoring force set by the **spring constant** 弹簧劲度系数 k. Its stored energy is $U = \frac{1}{2}kx^2$.

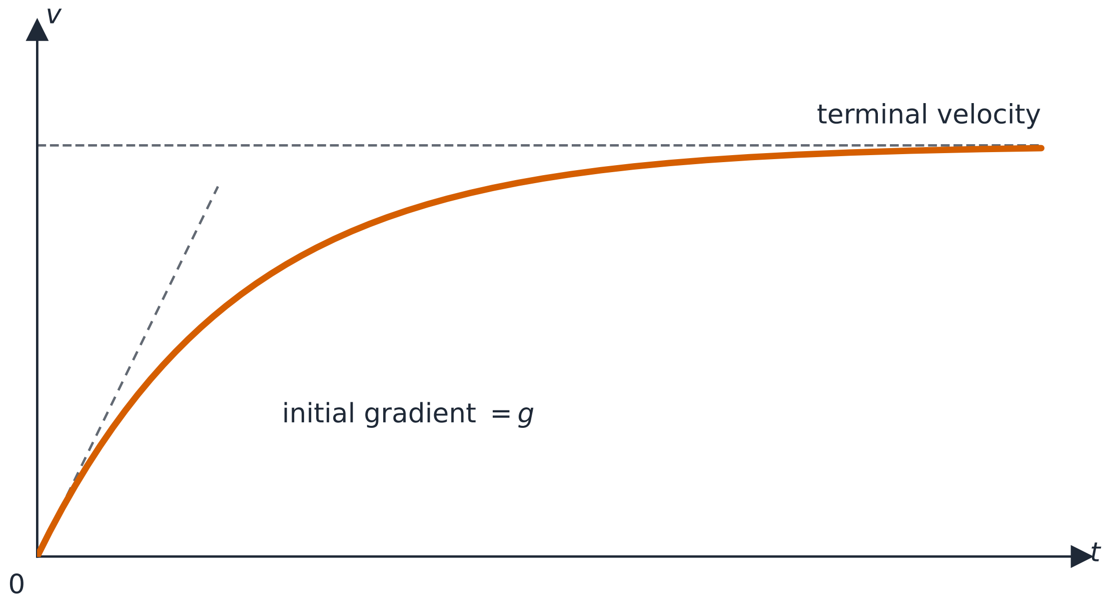


Hooke's law: extension is proportional to load up to the limit of proportionality

Resistive Forces

A **resistive force** 阻力 (drag) opposes motion through a fluid and **grows with speed**, often modeled as $F = -bv$ or $F = -cv^2$. Newton's second law then gives a differential equation, e.g. $m\dfrac{dv}{dt} = mg - bv$ for a falling object. As speed rises, drag builds until it balances the driving force; the object then stops accelerating and moves at a constant **terminal velocity** 终极速度, found by setting the net force (and dv/dt) to zero.

Worked example. For a falling object with $m\dfrac{dv}{dt} = mg - bv$, the terminal velocity is where $\dfrac{dv}{dt} = 0$: $mg = bv_T$, so $v_T = \dfrac{mg}{b}$. With $m = 0.10$ kg and $b = 0.50$ kg/s, $v_T = \dfrac{0.10(9.8)}{0.50} = 2.0$ m/s. (Solving the full equation gives $v(t) = v_T(1 - e^{-bt/m})$, approaching v_T exponentially.)



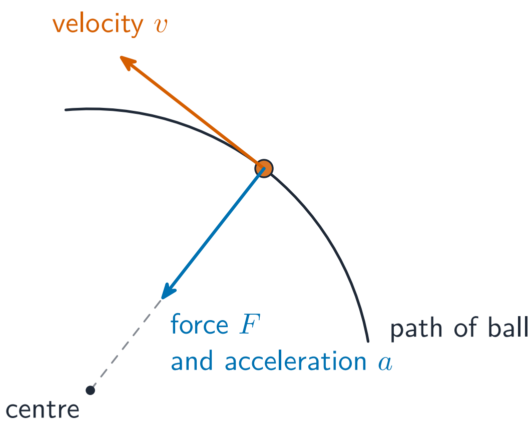
An object falling through a fluid speeds up to a terminal velocity

Circular Motion

Uniform circular motion has a **centripetal acceleration** 向心加速度 $a_c = \dfrac{v^2}{r}$ pointing to the center, requiring a **net inward force** $F_c = \dfrac{mv^2}{r}$ supplied by a real force (tension, gravity, friction, normal). There is no outward force. For vertical circles and banked curves, decompose the real forces to find which provides the centripetal requirement.

Worked example. A 0.50 kg ball is whirled on a 1.0 m string in a horizontal circle at 3.0 m/s. The string tension supplies the whole centripetal force: $T = \dfrac{mv^2}{r} = \dfrac{0.50(3.0)^2}{1.0} =$ 4.5 N.

Worked example (vertical circle). At the top of a vertical loop of radius r, gravity and the normal force *both* point toward the center: $N + mg = \dfrac{mv^2}{r}$. The slowest possible speed at the top is where the track pushes with nothing at all ($N = 0$): $v_{\min} = \sqrt{gr}$. For $r = 2.5$ m: $v_{\min} = \sqrt{9.8(2.5)} = 4.9$ m/s. Any slower and the cart leaves the track before the top.



The velocity points along the tangent; the centripetal force points to the centre

Exam tips

- Locate the **center of mass** with $x_{cm} = \frac{1}{M}\int x\,dm$ (or $\frac{\sum m_i x_i}{\sum m_i}$ for point masses) and use λ, σ, ρ for the mass element.
- The net external force moves the center of mass as if all mass sat there: $\vec{F}_{net} = M\vec{a}_{cm}$.
- Define your **system** clearly —internal forces cancel, so only external forces change its momentum.
- Exploit symmetry to shortcut a center-of-mass integral.
- Distinguish center of mass from center of gravity (identical in a uniform field).