Kinematics

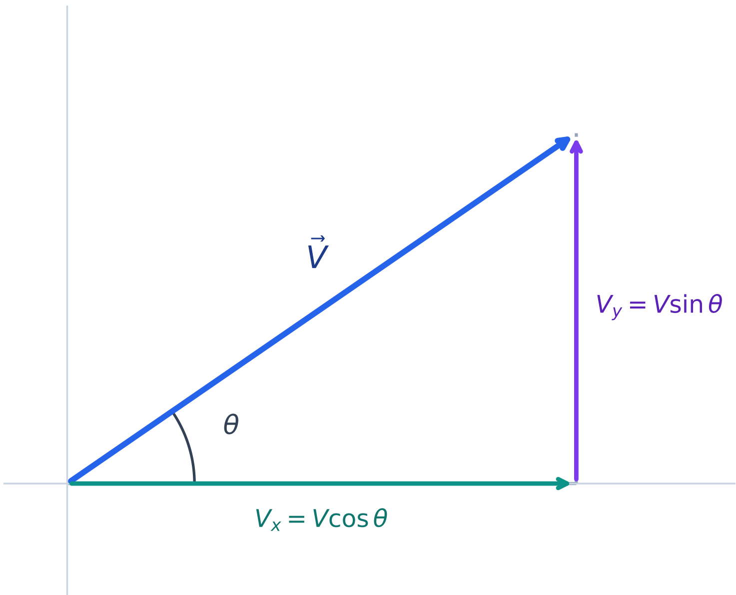
AP Physics C: Mechanics

Scalars and Vectors

Kinematics 运动学 describes motion. Two kinds of quantity:

- A **scalar** 标量 has only size (**magnitude** 大小): distance, speed, mass, time.
- A **vector** 矢量 has magnitude **and** direction: displacement, velocity, acceleration, force.

In this calculus-based course you resolve vectors into **components** 分量 and add them component by component; a vector's magnitude is $\sqrt{v_x^2 + v_y^2 + \cdots}$. Physics C also writes vectors with **unit vectors** 单位矢量: $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are directions of length one along x and y –handy because calculus can then act on each component separately.



Resolving a vector into its x and y components

Displacement, Velocity, and Acceleration

Because motion can change continuously, define the rates as **derivatives** 导数:

$$v = \frac{dx}{dt}, \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

Reversing this, integrate to recover velocity and position:

$$v(t) = v_0 + \int_0^t a\,dt, \qquad x(t) = x_0 + \int_0^t v\,dt.$$

Distinguish the *average* from the *instantaneous*: **average velocity** 平均速度 is $\Delta x/\Delta t$ over an interval, while **instantaneous velocity** 瞬时速度 is the derivative at one moment –the slope of the tangent line, and what a speedometer shows.

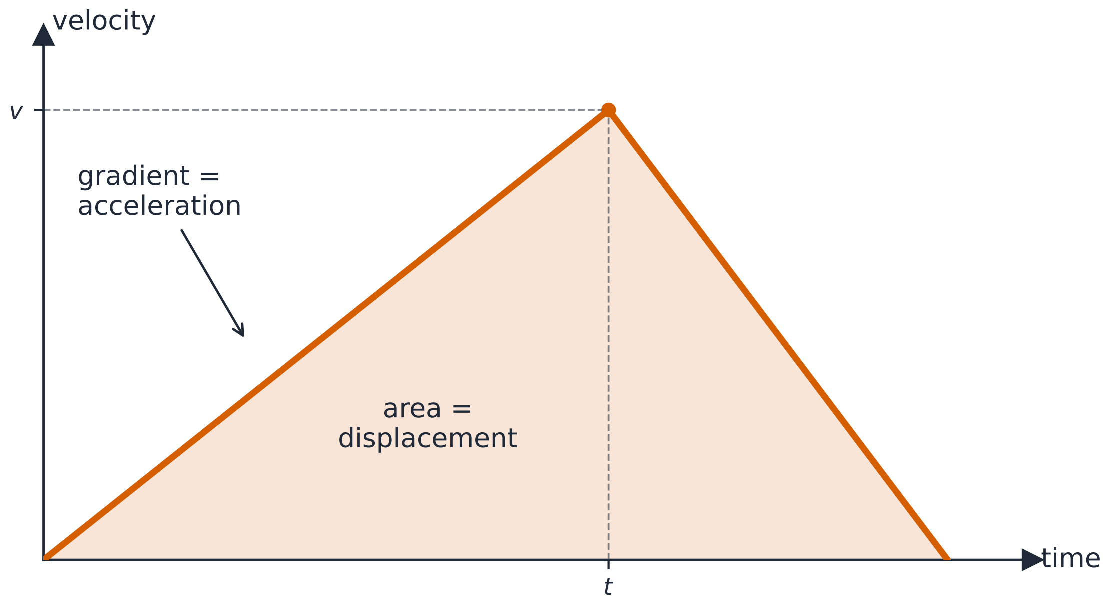
For **constant** acceleration these give the familiar kinematic equations ($v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\,\Delta x$). The most important constant-a case is **free fall** 自由落体: near Earth's surface every object, heavy or light, accelerates downward at $g = 9.8\ \text{m/s}^2$ once air resistance is negligible. When a or v varies with time, use the integrals directly.

Worked example. A particle moves with $x(t) = 2t^3 - 3t^2$ (metres). Differentiate for velocity and acceleration:

$$v = \frac{dx}{dt} = 6t^2 - 6t, \qquad a = \frac{dv}{dt} = 12t - 6.$$

At $t = 2$ s, $v = 24 - 12 = 12$ m/s and $a = 24 - 6 = 18\ \text{m/s}^2$. This calculus link –not just the constant-a formulas –is what distinguishes Physics C.

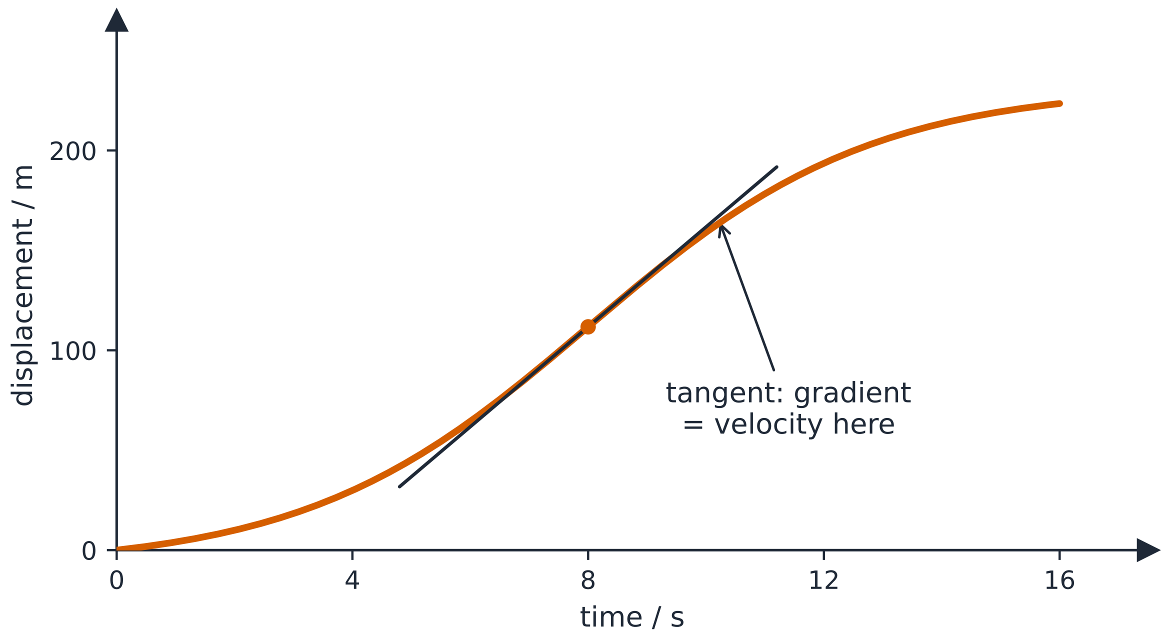
Worked example. A particle starts from rest at the origin with $a(t) = 6t$. Integrate: $v = \int 6t\,dt = 3t^2$ and $x = \int 3t^2\,dt = t^3$. At $t = 2$ s, $v = 12$ m/s and $x = 8$ m.



On a velocity-time graph the gradient is the acceleration and the area is the displacement

Representing Motion

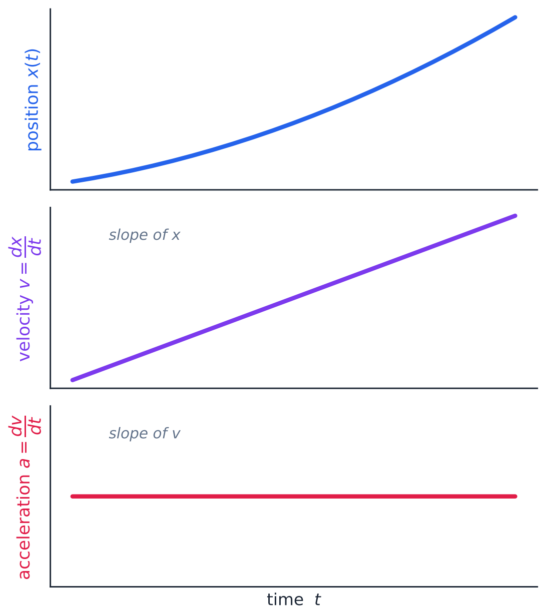
Move fluently between description, graph, table, and equation:



A displacement-time graph: the slope at any instant is the velocity

- On a **position–time** graph, the slope (dx/dt) is velocity.
- On a **velocity–time** graph, the slope is acceleration and the **area** $(\int v\,dt)$ is displacement.
- On an **acceleration–time** graph, the area $(\int a\,dt)$ is the change in velocity.

Slopes are derivatives; areas are integrals –the two are inverse operations here.



Position, velocity, and acceleration are linked by differentiation

Reference Frames and Relative Motion

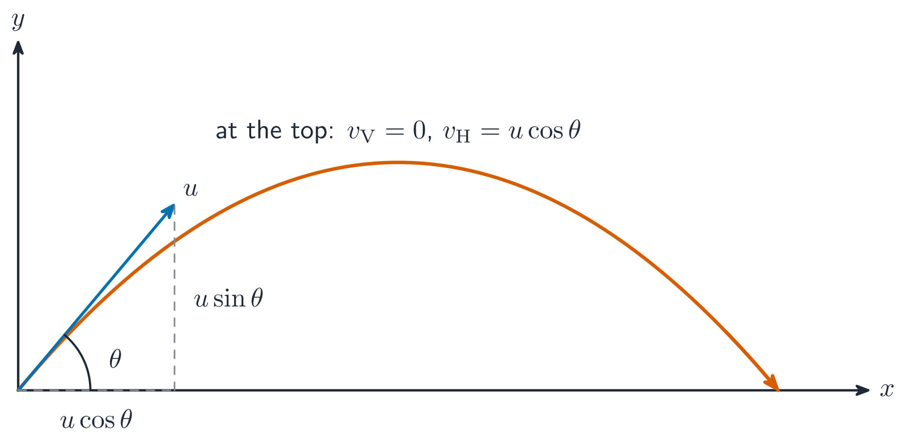
All motion is relative to a **reference frame** 参考系. Combine velocities by vector addition: $\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$. This handles boats crossing rivers and passengers on moving vehicles –**relative motion** 相对运动 problems. Read the subscripts as a chain: "A relative to B" plus "B relative to C" gives "A relative to C".

Worked example. A boat that moves at 4.0 m/s in still water heads straight across a river flowing at 3.0 m/s. Relative to the ground the boat moves at $\sqrt{4.0^2 + 3.0^2} = 5.0$ m/s, angled downstream. If the river is 80 m wide, the crossing still takes $t = \dfrac{80}{4.0} = 20$ s – only the across-stream component crosses the river; the current just carries the boat 60 m downstream.

Motion in Two or Three Dimensions

In two or three dimensions, position is a vector $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle$, and velocity and acceleration are its successive derivatives –each component handled independently.

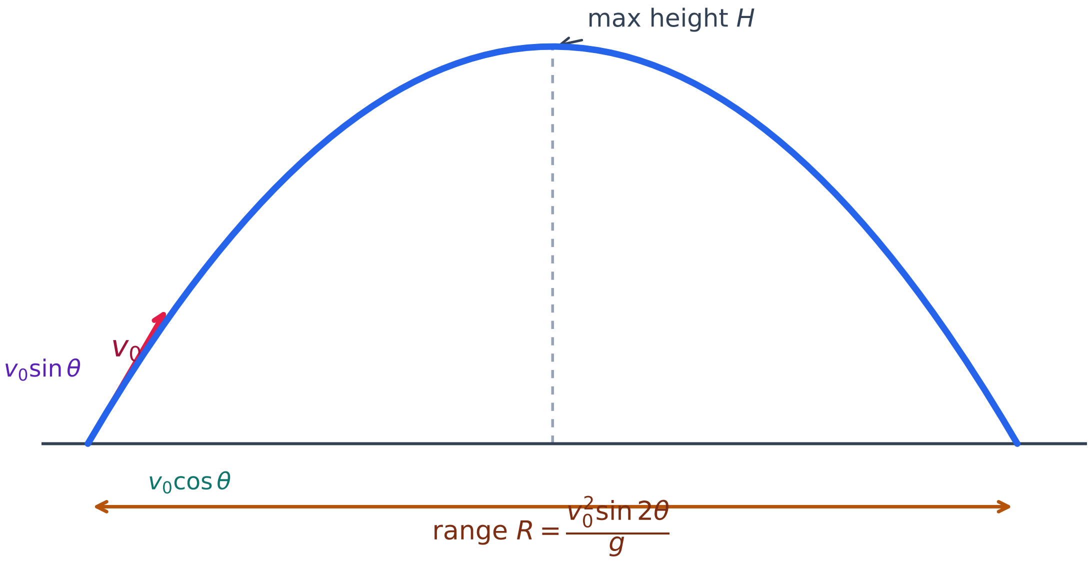
Worked example. $\vec{r}(t) = \left(2t^2\right)\hat{i} + \left(4t - t^3\right)\hat{j}$ (metres). Differentiating each component: $\vec{v} = 4t\,\hat{i} + (4 - 3t^2)\,\hat{j}$ and $\vec{a} = 4\,\hat{i} - 6t\,\hat{j}$. At $t = 1$ s: $\vec{v} = 4\hat{i} + 1\hat{j}$, so the speed is $\sqrt{17} \approx 4.1$ m/s –no new physics, just one derivative per component.



A projectile launched at an angle: the horizontal and vertical motions are independent

For **projectile motion** 抛体运动, horizontal and vertical motions are independent, linked only by time t: horizontally $a_x = 0$ (constant velocity), vertically $a_y = -g$. The path is a parabola. This component method extends to any two-dimensional motion where the accelerations along each axis are known.

Worked example. A ball is launched at 20 m/s, 30° above the horizontal ($g = 9.8$ m/s^2). The components are $v_{0x} = 20\cos 30° = 17.3$ m/s and $v_{0y} = 20\sin 30° = 10$ m/s. The vertical motion sets the time: total flight $= \dfrac{2v_{0y}}{g} = \dfrac{20}{9.8} = 2.0$ s, maximum height $= \dfrac{v_{0y}^2}{2g} = 5.1$ m, and range $= v_{0x} \times 2.0 = 35$ m.



A projectile launched at an angle: velocity components, maximum height, and range

Exam tips

- Resolve every vector into components before adding —never add magnitudes at an angle directly.
- On Physics C you are expected to use **calculus**: velocity is $\vec{v} = \frac{d\vec{r}}{dt}$ and acceleration $\vec{a} = \frac{d\vec{v}}{dt}$; reverse with integration.
- Carry and check **units** and treat direction with signs (choose a positive axis and stick to it).
- Use the dot product for work-type quantities and the cross product for torque and angular momentum.
- Sketch the vectors —a diagram catches sign and direction errors the algebra hides.