

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

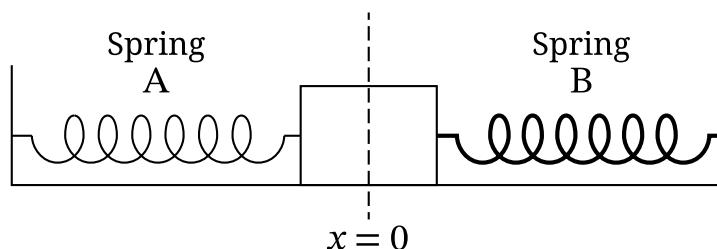


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

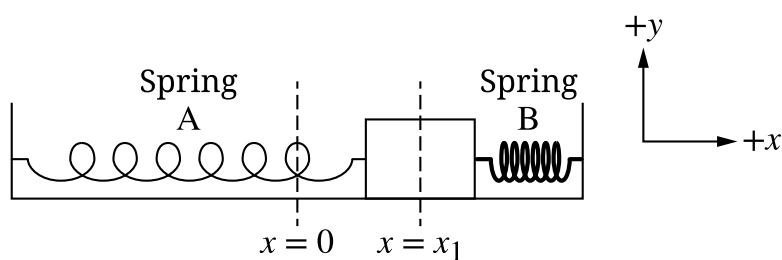


Figure 2

- A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.
- The height of the shaded bars should be proportional to the relative values of U_A , U_B , and K_{block} .
 - Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

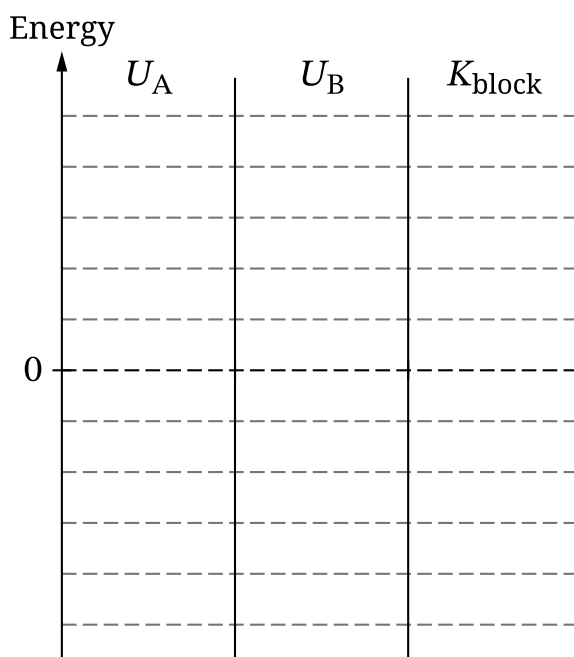
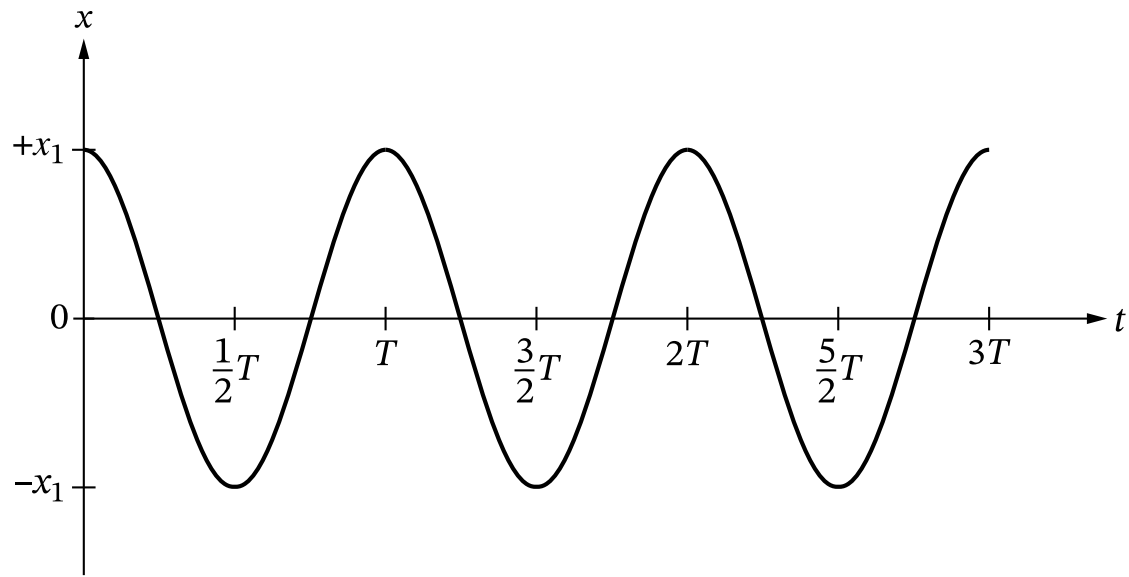


Figure 3

- B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m , k , x_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- C. In Scenario 1, the block oscillates with period T . The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

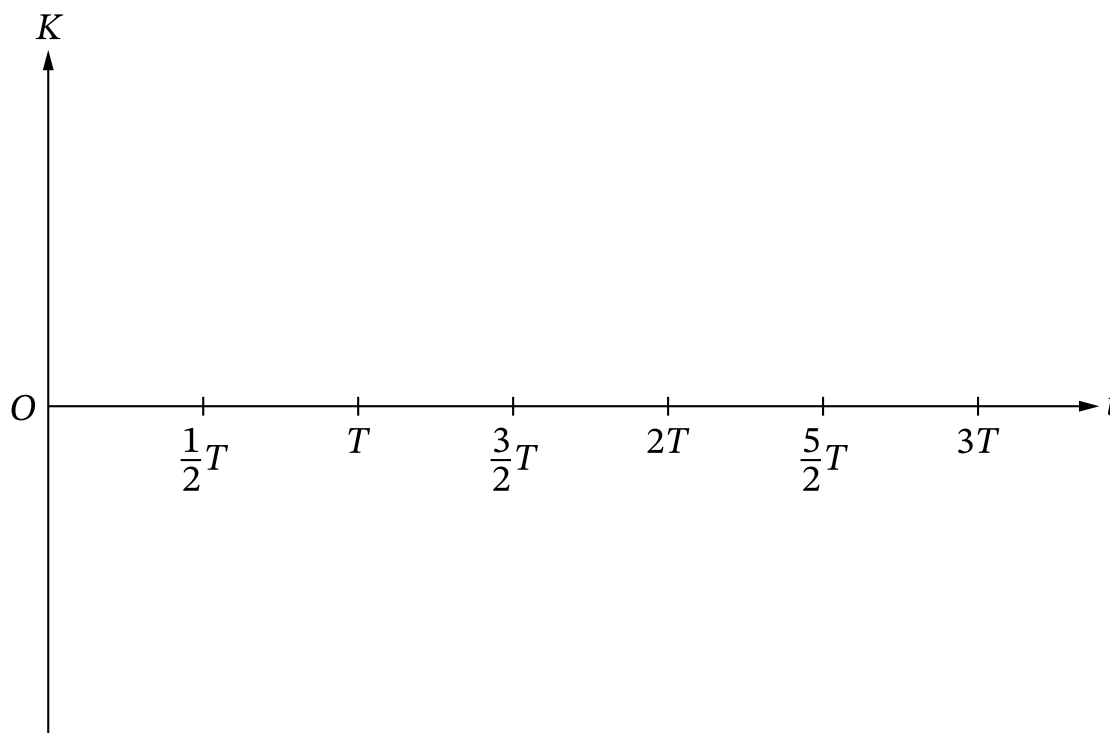


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

Figure 5

- D.** In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

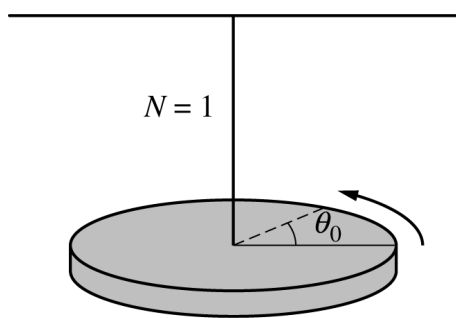


Figure 1

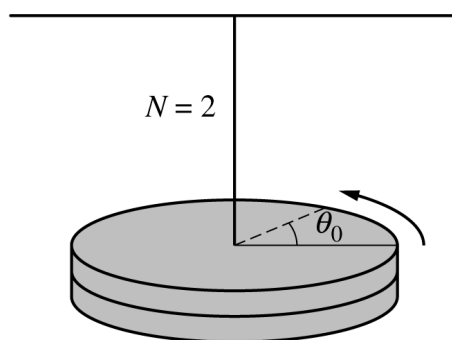


Figure 2

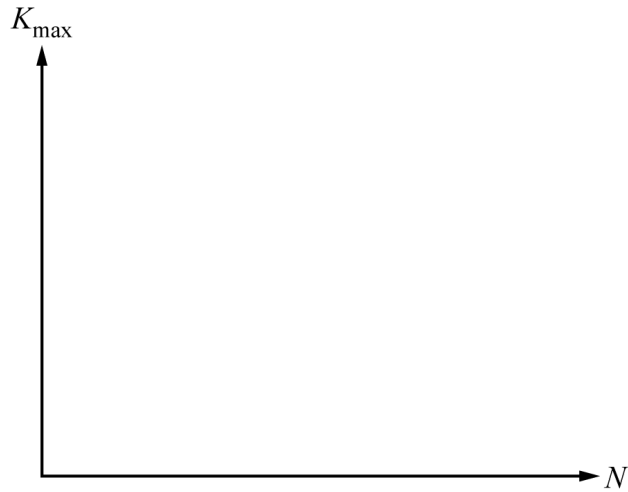
2. A student makes a torsional pendulum by suspending a uniform disk of mass M and radius R from a light wire with torsion constant κ that is attached to the center of the disk as shown in Figure 1. The rotational inertia of the disk is given by $I = \frac{1}{2}MR^2$. The student conducts an investigation to determine the relationship between the period of oscillation T of the torsional pendulum and the number N of identical disks that are suspended from the wire.

The student starts with a single disk. Holding the disk at a small initial angular displacement θ_0 from the untwisted position, the student releases the disk from rest and the pendulum oscillates. The student records the period of oscillation for a single disk. An additional identical disk is attached, as shown in Figure 2, and the procedure is repeated for $N = 2$ disks. This procedure is repeated through $N = 10$ identical disks. Assume the disks move together as one system.

- (a) Using $T = 2\pi\sqrt{\frac{I}{\kappa}}$, derive an expression for T as a function of N . Express your answer in terms of M , R , κ , N , and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

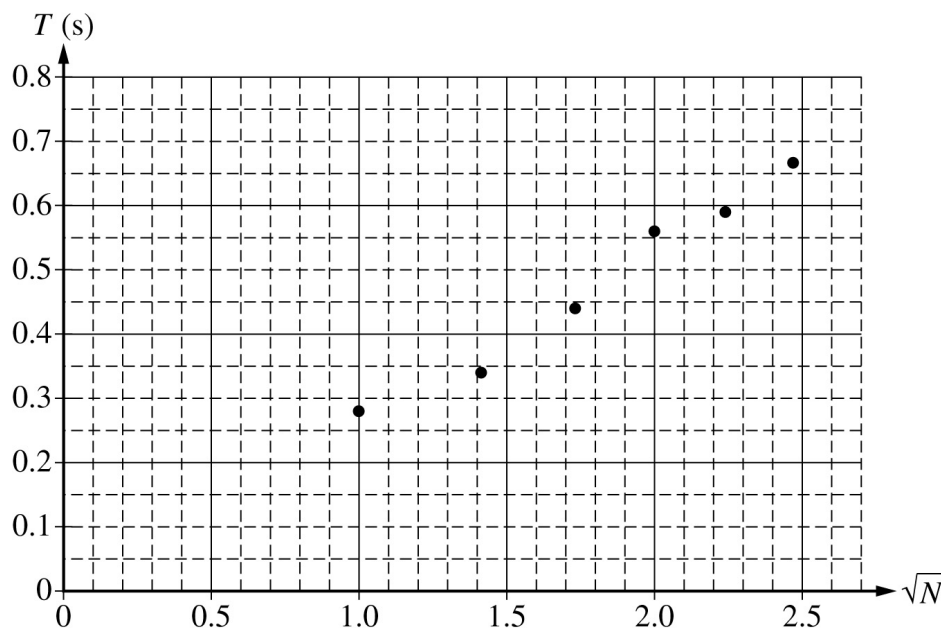
- (b) The potential energy stored in the torsional pendulum when the disks are displaced is $U = \frac{1}{2} \kappa (\Delta\theta)^2$. On the following axes, sketch a graph of the maximum kinetic energy $K_{\max}$ of the torsional pendulum as a function of N for $N \geq 1$.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(c) The student plots the data for T as a function of $\sqrt{N}$, as shown.



- i. Draw the best-fit line for the data.
- ii. The student previously determined that the radius of a disk is $R = 0.2$ m and found that $\kappa = 1.6$ N·m. Using the graph, calculate the mass M of a single disk.
- iii. The student finds that the value given by the manufacturer for the mass of the disk is less than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value of M . Justify your answer.

GO ON TO THE NEXT PAGE.

(d) The student repeats the experiment, but now the disks have a density that varies as a function of the radius of the disk according to $\rho = 0.3r$.

i. Would the slope of the best-fit line for this new data be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

___ greater than ___ less than ___ the same as

Justify your answer.

ii. When $N = 1$, the maximum angular speed of the torsional pendulum with a uniform disk is found to be ω_U . When $N = 1$, the maximum angular speed of the torsional pendulum with a nonuniform disk is $\omega_{\text{non-U}}$. Which of the following correctly compares ω_U and $\omega_{\text{non-U}}$?

___ $\omega_U > \omega_{\text{non-U}}$ ___ $\omega_U < \omega_{\text{non-U}}$ ___ $\omega_U = \omega_{\text{non-U}}$

Briefly justify your answer.

GO ON TO THE NEXT PAGE.

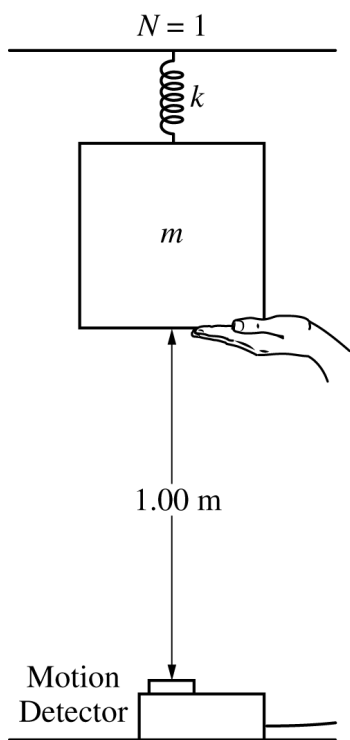


Figure 1

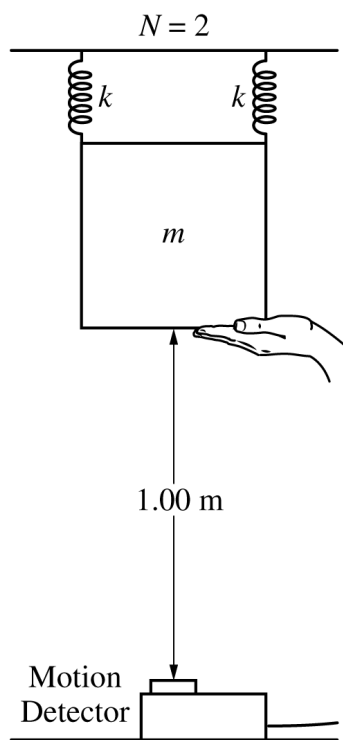


Figure 2

A student conducts an investigation to determine the relationship between the period of oscillation T of a system consisting of a block and N attached springs. The student starts with a block of mass m attached to a single ideal spring of spring constant k , as shown in Figure 1. The student holds the block so that the spring is neither stretched nor compressed at a vertical height 1.00 m above a motion detector. The student releases the block from rest and records the period of oscillation for the system consisting of the single spring and block. An additional identical spring is attached in parallel, as shown in Figure 2, and the procedure is repeated for $N = 2$ springs. This procedure is repeated through $N = 10$ springs.

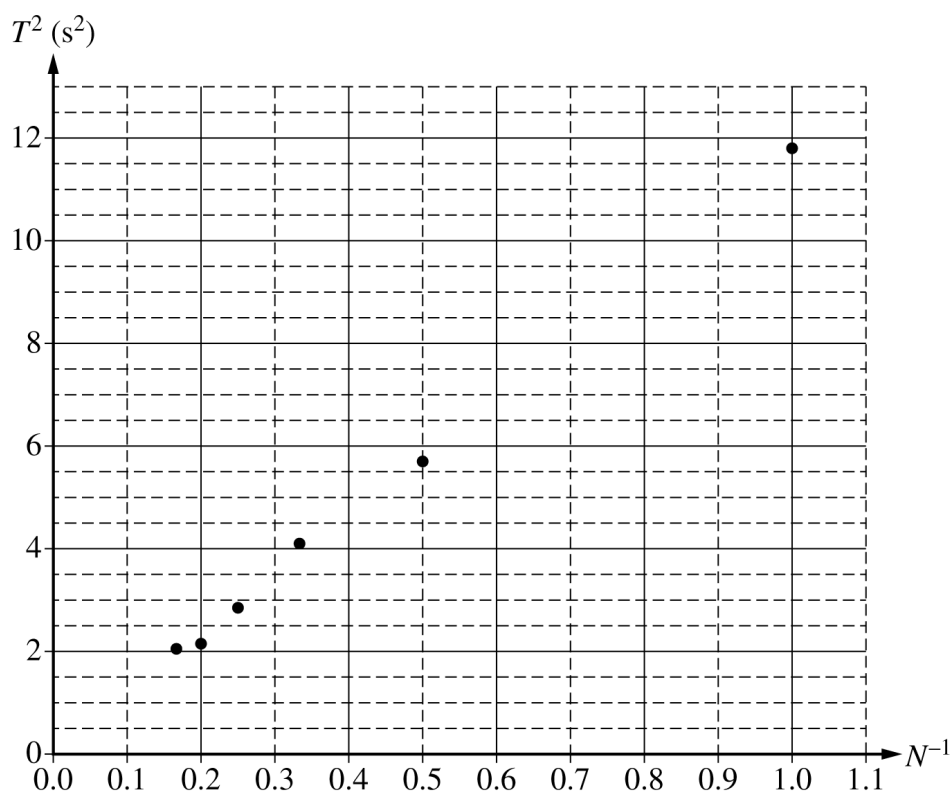
- (a) Derive an expression for T as a function of N . Express your answer in terms of m , k , N , and physical constants as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.(b) On the following axes, sketch a graph of T as a function of N for $N \geq 1$.**GO ON TO THE NEXT PAGE.**

Continue your response to QUESTION 2 on this page.

- (c) The student plots the data for T^2 as a function of N^{-1} , as shown.



- Draw the best-fit line for the data.
- The mass of the block is measured to be $m = 1.5$ kg. Using the graph, calculate an experimental value for the spring constant k for a single spring.
- The student finds that the value given by the manufacturer for the spring constant is larger than the value determined experimentally in part (c)(ii). Determine a single source of experimental error that could result in the observed difference in the value for k . Briefly justify your answer.

GO ON TO THE NEXT PAGE.

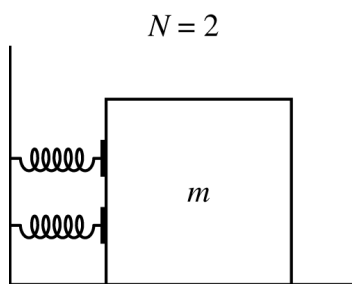
Continue your response to **QUESTION 2** on this page.

Figure 3

- (d) The student conducts a similar investigation to determine the relationship between the period of oscillation T of a system consisting of a block and a number N of identical springs but arranges the block-spring system horizontally on a table, as shown in Figure 3. Frictional forces between the table and the block are negligible. In each trial, the block is displaced the same horizontal distance from equilibrium and released from rest.

i. The student plots T^2 as a function of N^{-1} for this new data. Would the slope of the best-fit line from this new investigation be greater than, less than, or the same as the slope of the best-fit line in part (c)(i) ?

____ greater than ____ less than ____ the same as

Briefly justify your answer.

ii. When $N = 1$, the maximum speed of the block is found to be $v_{\max}$. When N increases, will $v_{\max}$ increase, decrease, or stay the same?

____ increase ____ decrease ____ stay the same

Justify your answer.

GO ON TO THE NEXT PAGE.